

2026 AMC 8 Exam

- 1 What is the value of the following expression?

$$1 + 2 - 3 + 4 + 5 - 6 + 7 + 8 - 9 + 10 + 11 - 12$$

- A. 18 B. 21 C. 24 D. 27 E. 30

Answer A

Solution Instead of calculating from left to right, we can group every three numbers together:

$$(1 + 2 - 3) + (4 + 5 - 6) + (7 + 8 - 9) + (10 + 11 - 12)$$

Now, let's calculate the value of each parenthesis (group) separately:

First Group:

$$1 + 2 - 3 = 3 - 3 = 0$$

Second Group:

$$4 + 5 - 6 = 9 - 6 = 3$$

Third Group:

$$7 + 8 - 9 = 15 - 9 = 6$$

Fourth Group:

$$10 + 11 - 12 = 21 - 12 = 9$$

Finally, we sum the results of these four groups:

$$0 + 3 + 6 + 9 = 18$$

Thus, the value of the expression is 18.

Correct Answer: (A)

- 2 In the array shown below, three 3s are surrounded by 2s, which are in turn surrounded by a border of 1s. What is the sum of the numbers in the array?

```

1  1  1  1  1  1  1
1  2  2  2  2  2  1
1  2  3  3  3  2  1
1  2  2  2  2  2  1
1  1  1  1  1  1  1

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A. 49

B. 51

C. 53

D. 55

E. 57

Answer C

Solution To solve this problem efficiently, we can count the quantity of each number (1s, 2s, and 3s) and calculate their sums separately.

Count the 3s: The problem states there are three 3s. The sum is $3 \times 3 = 9$.

Count the 2s: The 3s are in a central row. The 2s surround them immediately. There is a row of five 2s above, a row of five 2s below, and on the middle row, there is one 2 on the left and one 2 on the right. The total count of 2s is $5 + 5 + 1 + 1 = 12$. The sum is $12 \times 2 = 24$.

Count the 1s: The 1s form the outer border. There is a top row of seven 1s and a bottom row of seven 1s. In the three middle rows, there is one 1 on the far left and one 1 on the far right ($3 \times 2 = 6$). The total count of 1s is $7 + 7 + 6 = 20$. The sum is $20 \times 1 = 20$.

Finally, we add these sums together: $9 + 24 + 20 = 53$.

Correct Answer: (C) 53

3 Haruki has a piece of wire that is 24 centimeters long. He wants to bend it to form each of the following shapes, one at a time.

A regular hexagon with side length 5 cm.

A square of area 36 cm^2 .

A right triangle whose legs are 6 and 8 cm long.

Which of the shapes can Haruki make?

A. Triangle only

B. Hexagon and square only

C. Hexagon and triangle only

D. Square and triangle only

E. Hexagon, triangle, and square

Answer D

Solution To determine which shapes Haruki can make, the perimeter of the shape must be less than or equal to the length of the wire, which is **24** cm.

Regular Hexagon: A regular hexagon has **6** sides. With a side length of **5** cm, the perimeter is $6 \times 5 = 30$ cm. Since $30 > 24$, he cannot make the hexagon.

Square: The area is 36 cm^2 , so the side length is $\sqrt{36} = 6$ cm. The perimeter is $4 \times 6 = 24$ cm. Since $24 = 24$, he can make the square.

Right Triangle: The legs are **6** cm and **8** cm. Using the Pythagorean theorem ($a^2 + b^2 = c^2$), the hypotenuse is $\sqrt{6^2 + 8^2} = \sqrt{36 + 64} = \sqrt{100} = 10$ cm. The perimeter is the sum of the sides: $6 + 8 + 10 = 24$ cm. Since $24 = 24$, he can make the triangle.

Therefore, Haruki can make both the square and the triangle.

Correct Answer: D.

4 Brynn's savings decreased by **20%** in July, then increased by **50%** in August. Brynn's savings are now what percent of the original amount?

A. 80

B. 90

C. 100

D. 110

E. 120

Answer E

Solution Let the original savings be x .

July Decrease: The savings decreased by **20%**, meaning Brynn retained $100\% - 20\% = 80\%$ of the original amount. The new value is $0.8x$.

August Increase: The savings increased by **50%**. Note that this increase is applied to the new amount ($0.8x$), not the original. Multiplying by an increase of **50%** is the same as multiplying by **1.5** (since $100\% + 50\% = 150\% = 1.5$).

Calculation: The final amount is $0.8x \times 1.5$. Calculating the numbers: $0.8 \times 1.5 = 1.2$.

The final amount is $1.2x$, which is equal to **120%** of the original amount x .

Correct Answer: E.

5 Casey went on a road trip that covered 100 miles, stopping only for a lunch break along the way.

The trip took 3 hours in total and her average speed while driving was 40 miles per hour. In minutes, how long was the lunch break?

- A. 15 B. 30 C. 40 D. 45 E. 60

Answer B

Solution The total duration of the trip is the sum of the driving time and the break time.

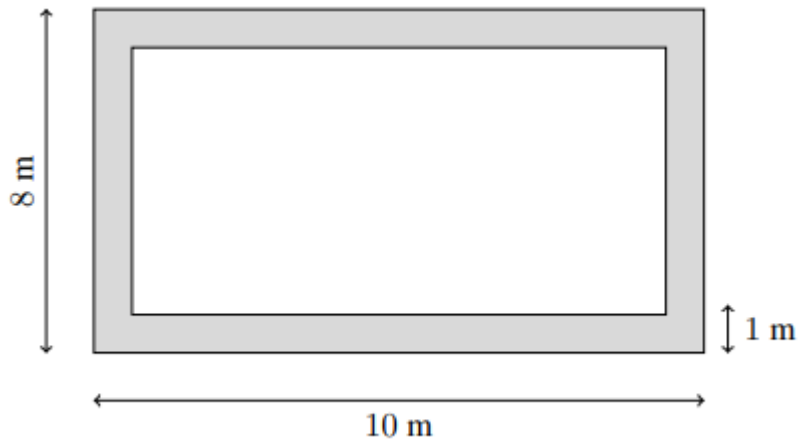
Find Driving Time: We use the formula $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$. The distance is 100 miles and the speed is 40 mph. So, the driving time is $\frac{100}{40} = 2.5$ hours.

Find Lunch Time: The total trip took 3 hours. Subtracting the driving time gives us the break time: $3 - 2.5 = 0.5$ hours.

Convert to Minutes: To find the minutes, we multiply the hours by 60: $0.5 \times 60 = 30$ minutes.

Correct Answer: B.

6 Peter lives near a rectangular field that is filled with blackberry bushes. The field is 10 meters long and 8 meters wide, and Peter can reach any blackberries that are within 1 meter of an edge of the field. The portion of the field he can reach is shaded in the figure below. What fraction of the area of the field can Peter reach?



- A. $\frac{1}{6}$ B. $\frac{1}{4}$ C. $\frac{1}{3}$ D. $\frac{3}{8}$ E. $\frac{2}{5}$

Answer E

Solution To find the fraction of the area Peter can reach, we need to calculate the area of the shaded region and divide it by the total area of the field.

Calculate Total Area: The field is a rectangle with length 10 m and width 8 m. The total area is $10 \times 8 = 80 \text{ m}^2$.

Calculate the Unreachable Area (Inner Rectangle): The shaded region is a border 1 meter wide along the edges. This creates an inner, unshaded rectangle. We find its dimensions by subtracting the border width (1 meter on each side) from the total dimensions.

Inner Length: $10 - 1 - 1 = 8 \text{ m}$.

Inner Width: $8 - 1 - 1 = 6 \text{ m}$.

Inner Area: $8 \times 6 = 48 \text{ m}^2$.

Calculate Shaded Area: The shaded area is the total area minus the unshaded inner area:
 $80 - 48 = 32 \text{ m}^2$.

Calculate the Fraction: The fraction of the field Peter can reach is $\frac{32}{80}$. To simplify this fraction, we divide the numerator and the denominator by their greatest common divisor, which is 16.

$$\frac{32 \div 16}{80 \div 16} = \frac{2}{5}$$

Correct Answer: E.

- 7 Mika would like to estimate how far she can ride a new model of electric bike on a fully charged battery. She completed two trips totaling 40 miles. The first trip used $\frac{1}{2}$ of the total battery power, while the second trip used $\frac{3}{10}$ of the total battery power. How many miles can this electric bike go on a fully charged battery?

A. 45 B. 48 C. 50 D. 52 E. 55

Answer C

Solution Let the total distance the bike can travel on a fully charged battery be x miles.

Calculate Total Battery Used: Mika used $\frac{1}{2}$ of the battery on the first trip and $\frac{3}{10}$ on the second:

$$\frac{1}{2} + \frac{3}{10} = \frac{5}{10} + \frac{3}{10} = \frac{8}{10}$$

Simplifying $\frac{8}{10}$ gives us $\frac{4}{5}$ of the total battery capacity.

Set Up Equation: We know that using $\frac{4}{5}$ of the battery allowed her to travel 40 miles. So, $\frac{4}{5}$ of the total distance x is equal to 40.

$$\frac{4}{5}x = 40$$

Solve for x : To find x , we divide 40 by $\frac{4}{5}$:

$$\begin{aligned} x &= 40 \times \frac{5}{4} \\ x &= \frac{200}{4} = 50 \end{aligned}$$

The bike can travel 50 miles on a fully charged battery.

Correct Answer: C.

- 8 A poll asked a number of people if they liked solving mathematics problems. Exactly 74% answered "yes." What is the fewest possible number of people who could have been asked the question?

A. 10

B. 20

C. 25

D. 50

E. 100

Answer D

Solution To find the fewest possible number of people, we need to express the percentage as a fraction in its simplest form.

Convert Percentage to Fraction:

$$74\% = \frac{74}{100}$$

Simplify the Fraction:

We simplify $\frac{74}{100}$ by dividing both the numerator and the denominator by their greatest common divisor. Both numbers are even, so we can divide by 2.

$$\frac{74 \div 2}{100 \div 2} = \frac{37}{50}$$

Analyze the Result: The number 37 is a prime number, so the fraction $\frac{37}{50}$ cannot be simplified any further. This means that for exactly 74% of the people to answer "yes," there must be at least 37 "yes" answers out of a total group of 50 people. If the group were smaller than 50, it would be impossible to get exactly 74% (for example, out of 25 people, each person represents 4%, so you could get 72% or 76%, but not 74%).

Therefore, the smallest possible total number of people is the denominator of the simplified fraction, which is 50.

Correct Answer: D.

9 What is the value of this expression?

$$\frac{\sqrt{16\sqrt{81}}}{\sqrt{81\sqrt{16}}}$$

A. $\frac{4}{9}$

B. $\frac{2}{3}$

C. 1

D. $\frac{3}{2}$

E. $\frac{9}{4}$

Answer B

Solution

To solve this problem, we need to simplify the numerator and the denominator separately, working from the innermost square roots outward.

Simplify the Numerator:

The expression is $\sqrt{16\sqrt{81}}$.

First, evaluate the inner square root: $\sqrt{81} = 9$.

Substitute this value back into the expression: $\sqrt{16 \times 9}$.

We can multiply the numbers first: $16 \times 9 = 144$.

Then, take the square root of the result: $\sqrt{144} = 12$.

Simplify the Denominator:

The expression is $\sqrt{81\sqrt{16}}$.

First, evaluate the inner square root: $\sqrt{16} = 4$.

Substitute this value back: $\sqrt{81 \times 4}$.

We can multiply the numbers first: $81 \times 4 = 324$.

Then, take the square root of the result: $\sqrt{324} = 18$.

(Alternatively: $\sqrt{81} \times \sqrt{4} = 9 \times 2 = 18$.)

Calculate the Final Fraction:

Now we divide the simplified numerator by the simplified denominator:

$$\frac{12}{18}$$

To simplify this fraction, divide both the numerator and the denominator by their greatest common divisor, which is 6:

$$\frac{12 \div 6}{18 \div 6} = \frac{2}{3}$$

Correct Answer: B.

10 Five runners completed the grueling Xmarathon: Luke, Melina, Nico, Olympia, and Pedro.

Nico finished 11 minutes behind Pedro.

Olympia finished 2 minutes ahead of Melina, but 3 minutes behind Pedro.

Olympia finished 6 minutes ahead of Luke.

Which runner finished fourth?

A. Luke

B. Melina

C. Nico

D. Olympia

E. Pedro

Answer A

Solution To determine the order of the runners, let's establish a timeline based on when they finished relative to one runner. Let's use Pedro as our reference point (0 minutes).

Pedro: Reference point. Time = P .

Nico: Finished 11 minutes behind Pedro. Time = $P + 11$.

Olympia: Finished 3 minutes behind Pedro. Time = $P + 3$.

Melina: Olympia finished 2 minutes ahead of Melina, which means Melina finished 2 minutes after Olympia. Time = $(P + 3) + 2 = P + 5$.

Luke: Olympia finished 6 minutes ahead of Luke, which means Luke finished 6 minutes after Olympia. Time = $(P + 3) + 6 = P + 9$.

Now, let's list their finish times relative to Pedro (smallest number means fastest time/earlier finish):

Pedro: +0 (1st)

Olympia: +3 (2nd)

Melina: +5 (3rd)

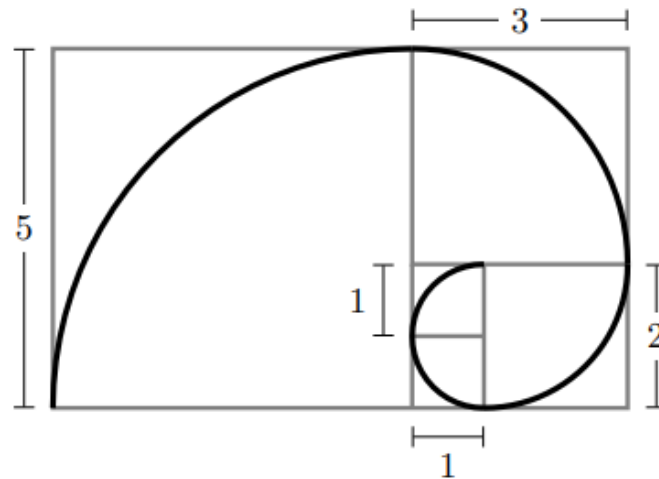
Luke: +9 (4th)

Nico: +11 (5th)

The runner who finished fourth is Luke.

Correct Answer: A.

- 11 Squares of side length 1, 1, 2, 3, and 5 are arranged to form the rectangle shown below. A curve is drawn by inscribing a quarter circle in each square and joining the quarter circles in order, from shortest to longest. What is the length of the curve?



- A. 4π B. 6π C. $\frac{13}{2}\pi$ D. 8π
 E. 13π

Answer B

Solution The curve is composed of five separate quarter-circle arcs. The length of a full circle's circumference is $C = 2\pi r$. Therefore, the length of a quarter circle is $\frac{1}{4} \times 2\pi r = \frac{\pi r}{2}$. We simply need to sum the lengths of these five arcs based on the side lengths of the squares (which correspond to the radii r):

First Arc: Radius $r = 1$. Length = $\frac{1\pi}{2}$.

Second Arc: Radius $r = 1$. Length = $\frac{1\pi}{2}$.

Third Arc: Radius $r = 2$. Length = $\frac{2\pi}{2} = \pi$.

Fourth Arc: Radius $r = 3$. Length = $\frac{3\pi}{2}$.

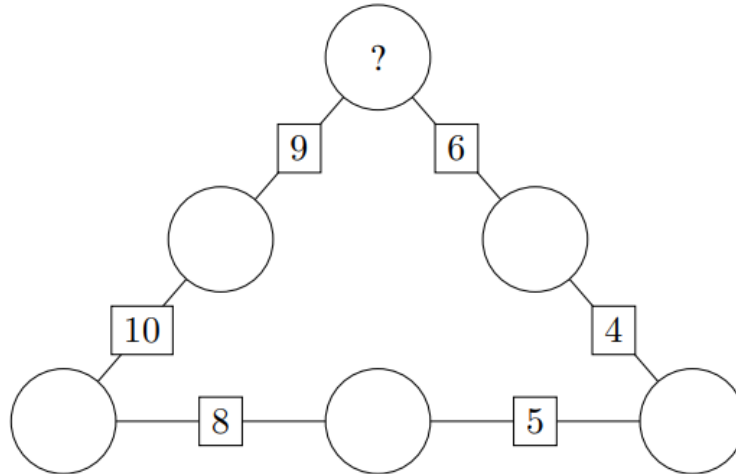
Fifth Arc: Radius $r = 5$. Length = $\frac{5\pi}{2}$.

Total Length Calculation:

$$\begin{aligned} \text{Total} &= \frac{\pi}{2} + \frac{\pi}{2} + \pi + \frac{3\pi}{2} + \frac{5\pi}{2} \\ \text{Total} &= \frac{\pi + \pi + 2\pi + 3\pi + 5\pi}{2} \\ \text{Total} &= \frac{12\pi}{2} \\ \text{Total} &= 6\pi \end{aligned}$$

Correct Answer: B.

- 12 In the figure below, each circle will be filled with a digit from 1 to 6. Each digit must appear exactly once. The sum of the digits in neighboring circles is shown in the box between them. What digit must be placed in the top circle?



- A. 2
B. 3
C. 4
D. 5
E. it is impossible to fill the circles

Answer D

Solution Let's label the circles to organize the information.

Top Row: T (Top)

Middle Row: ML (Middle Left), MR (Middle Right)

Bottom Row: BL (Bottom Left), BM (Bottom Middle), BR (Bottom Right)

We have the digits $\{1, 2, 3, 4, 5, 6\}$. Based on the connections in the diagram, we have these equations:

$$MR + BR = 4$$

$$BM + BR = 5$$

$$T + MR = 6$$

$$BL + BM = 8$$

$$T + ML = 9$$

$$ML + BL = 10$$

Step 1: Start with the most restrictive equation.

Look at equation (1): $MR + BR = 4$.

Since the digits must be distinct positive integers, the only pair that adds up to 4 is 1 and 3.

So, $\{MR, BR\}$ must be $\{1, 3\}$.

Step 2: Test the two possibilities.

Case A: Assume $MR = 3$ and $BR = 1$.

From equation (3): $T + MR = 6 \Rightarrow T + 3 = 6 \Rightarrow T = 3$.

This is impossible because digits must be unique, and we already used 3 for MR . So, this case is invalid.

Case B: Assume $MR = 1$ and $BR = 3$.

From equation (3): $T + MR = 6 \Rightarrow T + 1 = 6 \Rightarrow T = 5$.

From equation (2): $BM + BR = 5 \Rightarrow BM + 3 = 5 \Rightarrow BM = 2$.

From equation (5): $T + ML = 9 \Rightarrow 5 + ML = 9 \Rightarrow ML = 4$.

From equation (6): $ML + BL = 10 \Rightarrow 4 + BL = 10 \Rightarrow BL = 6$.

Step 3: Verify the final connection.

We have assigned all digits: $T = 5, MR = 1, BR = 3, BM = 2, ML = 4, BL = 6$.

We must check if the last unused equation (4) holds true:

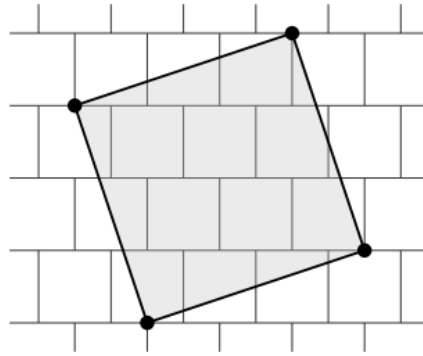
$$BL + BM = 8 \Rightarrow 6 + 2 = 8.$$

This is correct.

All conditions are satisfied, and the digit in the top circle (T) is 5.

Correct Answer: D.

- 13 The figure below shows a tiling of 1×1 unit squares. Each row of unit squares is shifted horizontally by half a unit relative to the row above it. A shaded square is drawn on top of the tiling. Each vertex of the shaded square is a vertex of one of the unit squares. In square units, what is the area of the shaded square?



- A. 10 B. $\frac{21}{2}$ C. $\frac{32}{3}$ D. 11
- E. $\frac{34}{3}$

Answer A

Solution To find the area of the shaded square, we need to determine the square of its side length.

We can do this by using the Pythagorean theorem.

The top left and the top right vertex are vertices of a right triangle with two legs **3** and **1**. So

the side length of the square is $\sqrt{1^2 + 3^2} = \sqrt{10}$.

Calculate Area:

The side length squared (s^2) is the sum of the squares of the horizontal and vertical changes:

$$s^2 = 3^2 + 1^2 = 9 + 1 = 10$$

The area of a square is equal to its side length squared. Therefore, the area is **10**.

Correct Answer: A. 10

- 14** Jami picked three equally spaced integer numbers on the number line. The sum of the first and the second numbers is **40**, while the sum of the second and third numbers is **60**. What is the sum of all three numbers?

- A. 70 B. 75 C. 80 D. 85 E. 90

Answer B

Solution Let the three numbers be a , b , and c . Because they are equally spaced, they form an arithmetic progression. This means the middle number is the average of the other two, or $a + c = 2b$.

Set up the Equations:

Equation 1: $a + b = 40$

Equation 2: $b + c = 60$

Find the Sum:

We want to find the total sum $S = a + b + c$.

If we add Equation 1 and Equation 2 together:

$$(a + b) + (b + c) = 40 + 60$$

$$a + 2b + c = 100$$

We can rearrange this sum as $(a + c) + 2b = 100$. Since $a + c = 2b$ (because they are equally spaced), we can substitute $2b$ for $(a + c)$:

$$2b + 2b = 100$$

$$4b = 100$$

$$b = 25$$

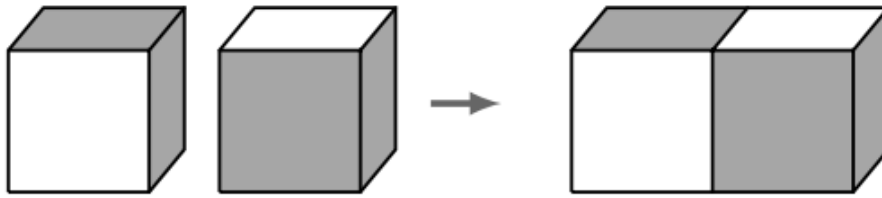
Calculate Final Sum:

$$\text{Sum} = (a + c) + b = 2b + b = 3b$$

$$\text{Sum} = 3 \times 25 = 75$$

Correct Answer: B.

15 Elijah has a large collection of identical wooden cubes which are white on 4 faces and gray on 2 faces that share an edge. He glues some cubes together face-to-face. The figure below shows 2 cubes being glued together, leaving 3 gray faces visible. What is the fewest number of cubes that he could glue together to ensure that no gray faces are visible, no matter how he rotates the figure?



- A. 4 B. 6 C. 8 D. 9 E. 27

Answer A

Solution To ensure no gray faces are visible, Elijah must construct a solid where every exterior face is white. This implies that all gray faces must be pointing inwards, glued against other faces.

Test the Smallest Option (4 cubes):

Imagine arranging 4 cubes in a $2 \times 2 \times 1$ square formation.

In this shape, each cube is a corner. A corner cube has 3 outer faces (visible) and 3 inner faces (hidden/glued to neighbors).

Since the 2 gray faces of a cube share an edge, we can always position the cube so that its 2 gray faces are hidden.

Since we can do this for all 4 cubes, the resulting object will have an all-white exterior.

Thus, the fewest number of cubes required is 4.

Correct Answer: A.

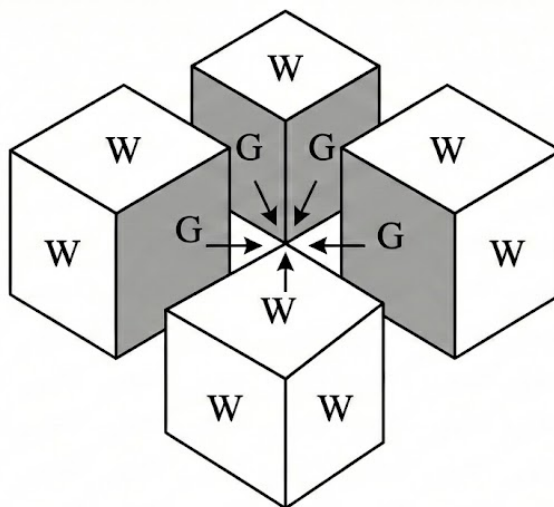


Figure for Problem 15: Four cubes arranged in a 2×2 square. All gray faces (G) are oriented inwards to be hidden, leaving only white faces (W) visible on the exterior.

16 Consider all positive four-digit integers consisting of only even digits. What fraction of these integers are divisible by 4?

- A. $\frac{1}{4}$ B. $\frac{2}{5}$ C. $\frac{1}{2}$ D. $\frac{3}{5}$ E. $\frac{3}{4}$

Answer D

Solution **Identify the Digits:** The problem specifies "only even digits". The available digits are $\{0, 2, 4, 6, 8\}$. Note that the first digit cannot be 0.

Divisibility Rule for 4: An integer is divisible by 4 if and only if the number formed by its last two digits is divisible by 4.

Analyze the Last Two Digits:

Tens Digit : This must be an even digit ($\{0, 2, 4, 6, 8\}$). Any even digit multiplied by 10 results in a multiple of 20. Since 20 is divisible by 4, tens digit can be any even numbers.

Units Digit : For the whole number to be divisible by 4, the units digit itself must be divisible by 4.

Count Valid Units Digits:

0 is divisible by 4. (Yes)

2 is not divisible by 4. (No)

4 is divisible by 4. (Yes)

6 is not divisible by 4. (No)

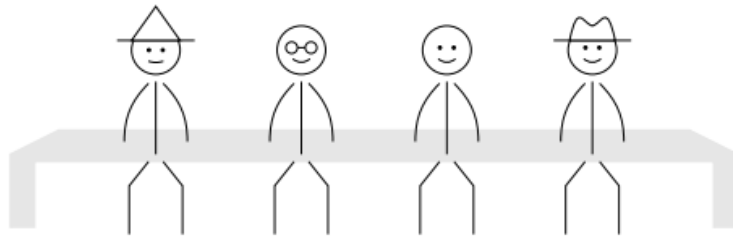
8 is divisible by 4. (Yes)

There are 3 valid choices (0, 4, 8) out of 5 possible choices.

$$\text{Fraction} = \frac{3}{5}$$

Correct Answer: D. $\frac{3}{5}$

17 Four students are seated in a row. They chat with the people sitting next to them, then rearrange themselves so that they are no longer seated next to any of the same people. How many rearrangements are possible?



- A. 2 B. 4 C. 9 D. 12 E. 24

Answer A

Solution Let the initial seating positions be **1, 2, 3, 4**. The pairs of neighbors in this arrangement are **(1, 2)**, **(2, 3)**, and **(3, 4)**. We are looking for a permutation of these four positions such that no two adjacent numbers in the new arrangement are consecutive integers.

Let's test the permutations systematically. Note that **1** cannot be next to **2**; **2** cannot be next to **1** or **3**; **3** cannot be next to **2** or **4**; **4** cannot be next to **3**.

Try starting with 1: The next number cannot be **2**.

If we start **1 – 3**: The remaining numbers are **2** and **4**.

1 – 3 – 2 – 4 and **1 – 3 – 4 – 2** are both **Invalid**.

If we start **1 – 4**: The remaining numbers are **2** and **3**. **1 – 4 – 2 – 3** and **1 – 4 – 3 – 2**: **(4, 3)** are both invalid.

Conclusion: No valid arrangement starts with **1**. By symmetry, no valid arrangement starts with **4**.

Try starting with 2: The next number cannot be **1** or **3**. It must be **4**.

Start **2 – 4**: Remaining are **1** and **3**.

2 – 4 – 1 – 3: **Valid**.

2 – 4 – 3 – 1: **Invalid**.

Try starting with 3: The next number cannot be **2** or **4**. It must be **1**.

Start **3 – 1**: Remaining are **2** and **4**.

3 – 1 – 4 – 2: **Valid**.

3 – 1 – 2 – 4: **Invalid**.

There are exactly **2** valid rearrangements: **2 – 4 – 1 – 3** and **3 – 1 – 4 – 2**.

Correct Answer: A.

18 In how many ways can 60 be written as the sum of two or more consecutive odd positive integers that are arranged in increasing order?

- A. 1 B. 2 C. 3 D. 4 E. 5

Answer B

Solution Let the sequence of k consecutive odd integers start with a . Since they are odd, the terms are $a, a + 2, a + 4, \dots$. This is an arithmetic progression with a common difference of 2.

The sum of an arithmetic progression is given by $\text{Sum} = \frac{k}{2}(2a + (k - 1)d)$.

Substituting $d = 2$ and $\text{Sum} = 60$:

$$60 = \frac{k}{2}(2a + (k - 1)2)$$

$$60 = k(a + k - 1)$$

Let $n = a + k - 1$. Then $k \times n = 60$.

Since a is an odd number, let $a = 2m + 1$.

$$\text{Then } n = (2m + 1) + k - 1 = 2m + k.$$

$$\text{Thus } k(2m + k) = 60$$

Notice that $(2m + k) - k = 2m$, which is an even number. This means that k and $(2m + k)$ differ by an even number, so they must have the same parity.

Since their product is 60 (an even number), they cannot both be odd. Therefore, both factors must be even.

We look for pairs of factors of 60 where both are even and $k < (2m + k)$ (since $a \geq 1$, the second factor is larger).

The factor pairs of 60 are:

1×60 (Mixed parity - Invalid)

2×30 (Both even - **Valid**)

3×20 (Mixed parity - Invalid)

4×15 (Mixed parity - Invalid)

5×12 (Mixed parity - Invalid)

6×10 (Both even - **Valid**)

There are exactly 2 valid cases.

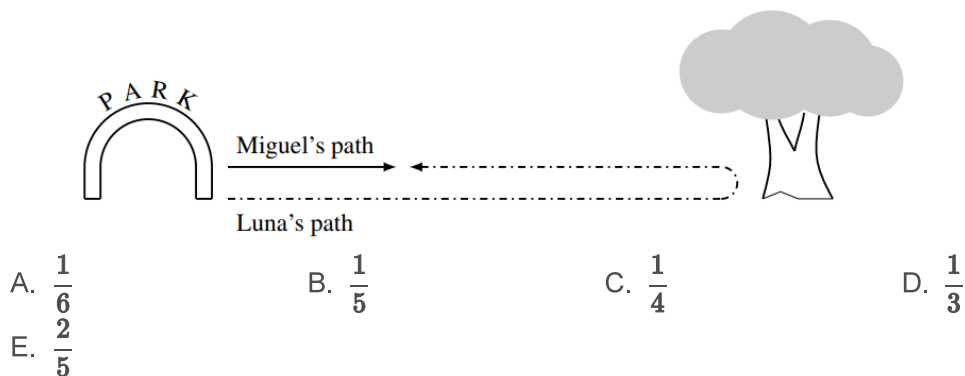
(For completeness:

Case 1: $k = 2$. Sum is $29 + 31 = 60$.

Case 2: $k = 6$. Sum is $5 + 7 + 9 + 11 + 13 + 15 = 60$.)

Correct Answer: B.

- 19 Miguel is walking with his dog, Luna. When they reach the entrance to a park, Miguel throws a ball straight ahead and continues to walk at a steady pace. Luna sprints toward the ball, which stops by a tree. As soon as the dog reaches the ball, she brings it back to Miguel. Luna runs 5 times faster than Miguel walks. What fraction of the distance between the entrance and the tree does Miguel cover by the time Luna brings him the ball?



Answer D

Solution Let v be Miguel's speed. Then Luna's speed is $5v$.

Let the distance from the entrance to the tree be D .

Since Miguel's speed : Luna's speed = $1 : 5$ and they start at the same time.

Length of Miguel's path : Length of Luna's path = $1 : 5$.

$$\text{Distance} = \frac{2D}{1+5} = \frac{D}{3}$$

Miguel covers $\frac{1}{3}$ of the total distance D .

Correct Answer: D. $\frac{1}{3}$

20 The land of Catania uses gold coins and silver coins. Gold coins are 1 mm thick and silver coins are 3 mm thick. In how many ways can Taylor make a stack of coins that is 8 mm tall using any arrangement of gold and silver coins, assuming order matters?

- A. 3 B. 7 C. 10 D. 13 E. 16

Answer D

Solution **Zero Silver Coins:**

We must use eight 1s.

Arrangements: Only 1 way.

One Silver Coin:

We use one 3 and five 1s ($3 + 1 \times 5 = 8$).

Total coins: 6.

We need to arrange five 1s and one 3. The 3 can be in any of the 6 positions.

Arrangements: 6 ways.

Two Silver Coins:

We use two 3s and two 1s ($3 \times 2 + 1 \times 2 = 8$).

Total coins: 4.

We need to arrange two 3s and two 1s. The number is $4C_2 = 6$.

Arrangements: 6 ways.

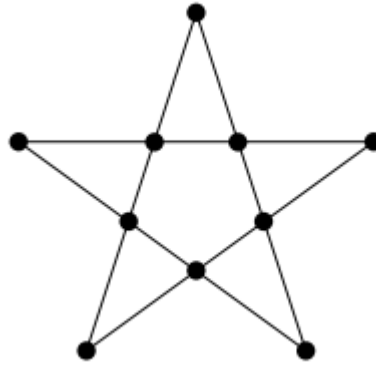
Total Ways:

Summing the possibilities from each case:

$$1 + 6 + 6 = 13$$

Correct Answer: D. 13

- 21 Charlotte the spider is walking along a web shaped like a 5-pointed star, shown in the figure below. The web has 5 outer points and 5 inner points. Each time Charlotte reaches a point, she randomly chooses a neighboring point and moves to that point. Charlotte starts at one of the outer points and makes 3 moves (re-visiting points is allowed). What is the probability she is now at one of the outer points?



- A. $\frac{1}{5}$
 B. $\frac{1}{4}$
 C. $\frac{2}{5}$
 D. $\frac{1}{2}$
 E. $\frac{3}{5}$

Answer B

Solution Let's track Charlotte's position at each step.

There are two types of points: Outer (O) and Inner (I).

Outer Points: Each outer point connects only to 2 inner points. (Degree = 2).

Inner Points: Each inner point connects to 2 outer points and 2 other inner points. (Degree = 4).

Charlotte starts at an **Outer** point.

Move 1: From an Outer point, the only neighbors are Inner points. So, with probability 1, she moves to an **Inner** point.

Current State: **Inner** (Probability: 100%)

Move 2: She is at an Inner point. She has 4 neighbors (2 Outer, 2 Inner). She chooses randomly.

Probability of moving to **Outer**: $\frac{2}{4} = \frac{1}{2}$.

Probability of moving to **Inner**: $\frac{2}{4} = \frac{1}{2}$.

Current State:

At **Outer** with probability $\frac{1}{2}$.

At **Inner** with probability $\frac{1}{2}$.

Move 3: We analyze the two possibilities from step 2 separately.

Case A (She was at Outer): Probability was $\frac{1}{2}$. From Outer, she must move to Inner.

Result: Ends at **Inner**. (Contribution to probability: $\frac{1}{2} \times 1 = \frac{1}{2}$ for Inner).

Case B (She was at Inner): Probability was $\frac{1}{2}$. From Inner, she chooses randomly again (2 Outer, 2 Inner).

Moves to **Outer**: Probability $\frac{1}{2}$. (Contribution: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ for Outer).

Moves to **Inner**: Probability $\frac{1}{2}$. (Contribution: $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ for Inner).

Final Calculation:

The only way to end at an Outer point is via Case B.

Probability of ending at Outer = $\frac{1}{4}$.

Correct Answer: B. $\frac{1}{4}$

22 The integers from 1 through 25 are arbitrarily separated into five groups of 5 numbers each. The median of each group is identified. Let M equal the median of the five medians. What is the least possible value of M ?

A. 9

B. 10

C. 12

D. 13

E. 14

Answer A

Solution To find the least possible value of the median of medians (M), we need to make M as small as possible.

Let the five group medians be m_1, m_2, m_3, m_4, m_5 .

If we order these medians from smallest to largest, M will be the 3rd one (the middle one).

For M to be the median of the medians, there must be at least 3 groups whose medians are less than or equal to M . Let's call these groups G_1, G_2, G_3 .

For a number to be the median of a group of 5, it must be greater than or equal to at least 3 numbers in that group (including itself).

In G_1 , there are 3 numbers $\leq m_1$. Since $m_1 \leq M$, these 3 numbers are $\leq M$.

In G_2 , there are 3 numbers $\leq m_2$. Since $m_2 \leq M$, these 3 numbers are $\leq M$.

In G_3 , there are 3 numbers $\leq m_3$. Since $m_3 \leq M$, these 3 numbers are $\leq M$.

Therefore, there must be at least $3 + 3 + 3 = 9$ distinct numbers in the entire set of 25 integers that are less than or equal to M .

The smallest 9 positive integers are 1, 2, 3, 4, 5, 6, 7, 8, 9.

For there to be 9 numbers less than or equal to M , M must be at least 9.

We can confirm this is possible with an example distribution:

Group 1: {1, 2, 3, 24, 25} (Median = 3)

Group 2: {4, 5, 6, 22, 23} (Median = 6)

Group 3: {7, 8, 9, 20, 21} (Median = 9)

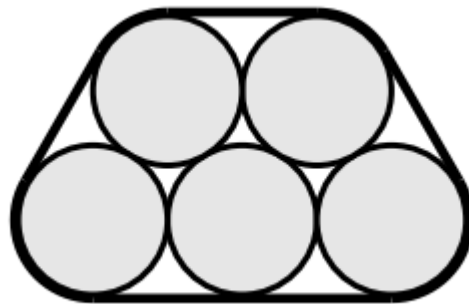
Group 4: {10, 11, 12, 18, 19} (Median = 12)

Group 5: {13, 14, 15, 16, 17} (Median = 15)

The medians are 3, 6, 9, 12, 15. The median of these is 9.

Correct Answer: A.

- 23 Lakshmi has 5 round coins of diameter 4 centimeters. She arranges the coins in 2 rows on a table top, as shown below, and wraps an elastic band tightly around them. In centimeters, what will be the length of the band?



- A. $2\pi + 20$ B. $\frac{5}{2}\pi + 20$
 C. $4\pi + 20$ D. $\frac{9}{2}\pi + 20$
 E. $5\pi + 20$

Answer C

Solution The length of a band wrapped around a collection of identical circles is equal to the **perimeter of the polygon formed by the centers of the outer circles** plus the **circumference of one circle**.

Calculate Perimeter of the Polygon of Centers:

The shape formed by the centers is an isosceles trapezoid.

Bottom edge length: 8 cm.

Top edge length: 4 cm.

Left edge length: 4 cm.

Right edge length: 4 cm.

Total Perimeter = $8 + 4 + 4 + 4 = 20$ cm. This corresponds to the straight sections of the band.

Calculate the Curved Sections:

The curved parts of the band at the corners always sum up to the circumference of exactly one circle.

Circumference $C = \pi \times \text{diameter} = 4\pi$.

Total Length:

Length = Straight Sections + Curved Sections

Length = $20 + 4\pi$.

Correct Answer: C.

24 The notation $n!$ (read " n factorial") is defined as the product of the first n positive integers. (For example, $3! = 1 \cdot 2 \cdot 3 = 6$.) Define the superfactorial of a positive integer, denoted by $n^!$, to be the product of the factorials of the first n integers. (For example, $3^! = 1! \cdot 2! \cdot 3! = 12$.) How many factors of 7 appear in the prime factorization of $51^!$, the superfactorial of 51?

- A. 147 B. 150 C. 156 D. 168 E. 171

Answer E

Solution We want to find the exponent of 7 in the prime factorization of $51! = 1! \times 2! \times 3! \times \cdots \times 51!$.

This is equivalent to summing the number of factors of 7 in each factorial $k!$ for $k = 1$ to 51.

Part 1:

For $k = 1$ to 6, value is 0.

For $k = 7$ to 13 (7 numbers), value is 1. Sum: $7 \times 1 = 7$.

For $k = 14$ to 20 (7 numbers), value is 2. Sum: $7 \times 2 = 14$.

...

For $k = 42$ to 48 (7 numbers), value is 6. Sum: $7 \times 6 = 42$.

For $k = 49$ to 51 (3 numbers), value is 7. Sum: $3 \times 7 = 21$.

Sum Part 1 = $7(1 + 2 + 3 + 4 + 5 + 6) + 21$

Sum Part 1 = $7(21) + 21 = 147 + 21 = 168$.

49 is multiplied by two 7s, but we calculate it only one time in part 1, so we need to add them in part 2.

Part 2:

For $k = 1$ to 48, value is 0.

For $k = 49$ to 51 (3 numbers), value is 1.

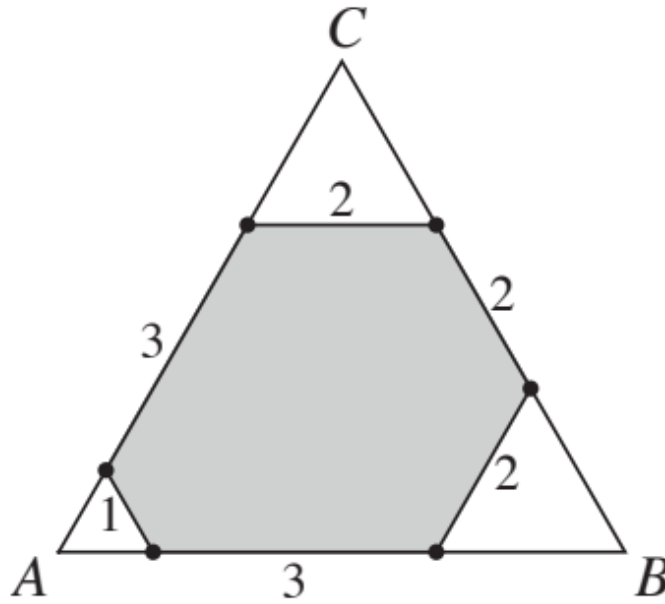
Sum Part 2 = $3 \times 1 = 3$.

Total Sum:

$168 + 3 = 171$.

Correct Answer: E. 171

25 In an equiangular hexagon, all interior angles measure 120° . An example of such a hexagon with side lengths of 2, 3, 1, 3, 2, and 2 is shown below, inscribed in equilateral triangle ABC . Consider all equiangular hexagons with positive integer side lengths that can be inscribed in $\triangle ABC$, with all six vertices on the sides of the triangle. What is the total number of such hexagons? Hexagons that differ only by a rotation or a reflection are considered the same.



- A. 4 B. 5 C. 6 D. 7 E. 8

Answer E

Solution To solve this, we first determine the properties of the equilateral triangle ABC that circumscribes the hexagon.

The hexagon cuts off three smaller triangles at the corners A , B , and C . Since the hexagon is equiangular (interior angles of 120°), the exterior angles are 60° . Thus, the three small triangles at the corners are equilateral triangles.

Let the side lengths of these small corner triangles be x , y , and z at corners A , B , and C respectively.

The side lengths of the hexagon alternate between the sides of these corner triangles (x, y, z) and the segments along the sides of the large triangle that connect them.

Let the side lengths of the corner triangles be x, y, z . These correspond to three non-adjacent sides of the hexagon.

The other three sides of the hexagon are formed by the remaining parts of the triangle's sides. Their lengths are $6 - x - y$, $6 - y - z$, and $6 - z - x$.

The problem states that all hexagon side lengths must be positive integers.

This gives us the following conditions:

$$x, y, z \geq 1$$

$$6 - x - y \geq 1 \Rightarrow x + y \leq 5$$

$$6 - y - z \geq 1 \Rightarrow y + z \leq 5$$

$$6 - z - x \geq 1 \Rightarrow z + x \leq 5$$

Since hexagons that differ only by rotation or reflection are considered the same, we are looking for the number of unique multisets $\{x, y, z\}$ that satisfy the conditions. The order of x, y, z does not matter because we can rotate or reflect the triangle to permute the corners. Let's list the valid triplets $\{x, y, z\}$ assuming $1 \leq x \leq y \leq z$ to avoid duplicates. The condition $y + z \leq 5$ is the most restrictive (since z is the largest).

If $z = 1$:

y must be 1.

x must be 1.

Set: $\{1, 1, 1\}$. (Sum of largest pair: $1 + 1 = 2 \leq 5$. Valid).

If $z = 2$:

y can be 1 or 2.

If $y = 1$, $x = 1$. Set: $\{1, 1, 2\}$. (Sum: $1 + 2 = 3 \leq 5$. Valid).

If $y = 2$, x can be 1 or 2.

Set: $\{1, 2, 2\}$. (Sum: $2 + 2 = 4 \leq 5$. Valid).

Set: $\{2, 2, 2\}$. (Sum: $2 + 2 = 4 \leq 5$. Valid).

If $z = 3$:

y can be 1 or 2 (since $y + 3 \leq 5 \Rightarrow y \leq 2$).

If $y = 1$, $x = 1$. Set: $\{1, 1, 3\}$. (Sum: $1 + 3 = 4 \leq 5$. Valid).

If $y = 2$, x can be 1 or 2.

Set: $\{1, 2, 3\}$. (Sum: $2 + 3 = 5 \leq 5$. Valid).

Set: $\{2, 2, 3\}$. (Sum: $2 + 3 = 5 \leq 5$. Valid).

If $z = 4$:

y must be 1 (since $y + 4 \leq 5 \Rightarrow y \leq 1$).

x must be 1.

Set: $\{1, 1, 4\}$. (Sum: $1 + 4 = 5 \leq 5$. Valid).

If $z = 5$:

y must be ≤ 0 , which is impossible.

Counting the valid sets, we have:

$\{1, 1, 1\}$

$\{1, 1, 2\}$

$\{1, 2, 2\}$

$\{2, 2, 2\}$

$\{1, 1, 3\}$

$\{1, 2, 3\}$

$\{2, 2, 3\}$

$\{1, 1, 4\}$

There are exactly 8 such hexagons.

Correct Answer: E. 8