

2016 AMC 8

- 1 (1分) The longest professional tennis match lasted a total of 11 hours and 5 minutes. How many minutes was that? () .
- A. 605 B. 655 C. 665 D. 1005 E. 1105

Answer C

Solution 方法一：职业网球比赛单场最长记录为11小时5分钟。换算成分钟，总共是多少？

最好把11个小时五分钟分成两部分，一个11小时，一个是五分钟。我们知道一个小时是60分钟，所以是11个小时是 $11 \cdot 60 = 660$ 分钟，加上五分钟我们得到 $660 + 5 = 665$ 。

故选C。

方法二：It is best to split 11 hours and 5 minutes into 2 parts, one of 11 hours and another of 5 minutes. We know that there is 60 minutes in a hour. Therefore, there are $11 \cdot 60 = 660$ minutes in 11 hours. Adding the second part (the 5 minutes) we get $660 + 5 = \boxed{(C)665}$.

- 2 (1分) In rectangle $ABCD$, $AB = 6$ and $AD = 8$. Point M is the midpoint of $\overline{AD}$. What is the area of $\triangle AMC$? () .
- A. 12 B. 15 C. 18 D. 20 E. 24

Answer A

Solution 方法一：矩形 $ABCD$ 中， $AB = 6$ ， $AD = 8$ 。点 M 是 $\overline{AD}$ 的中点。求 $\triangle AMC$ 的面积。

使用三角形的面积公式： $A = \frac{bh}{2}$ ，其中 A 是面积， b 是底， h 是高，给出方程

$$A = \frac{4 \cdot 6}{2} = \frac{24}{2} = 12.$$

故选A。

方法二：与矩形的高和底相等的三角形面积是矩形的面积的一半，这意味着底的一半，相同的高的三角形面积是矩形面积的四分之一，所以我们有 $\frac{48}{4} = 12$.

故选A .

方法三：Use the triangle area formula for triangles: $A = \frac{bh}{2}$, where A is the area, b is the base, and h is the height. This equation gives us $A = \frac{4 \cdot 6}{2} = \frac{24}{2} = 12$.

方法四：A triangle with the same height and base as a rectangle is half of the rectangle's area. This means that a triangle with half of the base of the rectangle and also the same height means its area is one quarter of the rectangle's area. Therefore, we get $\frac{48}{4} = 12$.

3 (1分) Four students take an exam. Three of their scores are 70, 80 and 90. If the average of their four scores is 70, then what is the remaining score? () .

- A. 40 B. 50 C. 55 D. 60 E. 70

Answer A

Solution 方法一：四名学生参加一场考试 . 其中三人分数为70 , 80 , 90 . 若四人平均分为70 , 则第四位学生的分数是 ?

我们可以称剩下的分数为 t , 我们也知道平均分是 $70 = \frac{70 + 80 + 90 + r}{4}$, 我们可以利用基本代数来求解 r : $\frac{70 + 80 + 90 + r}{4} = 70$, $\frac{240 + r}{4} = 70$, $240 + r = 280$, $r = 40$, 所以答案是 40 .

故选A .

方法二：We can call the remaining score r . We also know that the average, 70, is equal to $\frac{70 + 80 + 90 + r}{4}$.

We can use basic algebra to solve for r : $\frac{70 + 80 + 90 + r}{4} = 70$

$$\frac{240 + r}{4} = 70$$

$$240 + r = 280$$

$r = 40$ giving us the answer of **(A)40**.

4 (1分) When Cheenu was a boy he could run 15 miles in 3 hours and 30 minutes. As an old man he can now walk 10 miles in 4 hours. How many minutes longer does it take for him to travel a mile now compared to when he was a boy? () .

- A. 6 B. 10 C. 15 D. 18 E. 30

Answer B

Solution 方法一：当 Chennu 是个小男孩时他能在3小时30分跑完15英里。当他老了他能在4小时走完10英里。他年老与年幼时相比，走1英里要多花多少分钟？

当Cheenu还是个男孩的时候，他能够用三小时30分钟跑15英里，时间= $3 \times 60 + 30$ 分钟，速度为 $\frac{210}{15} = 14$ 分钟每英里。当他是大人的时候，他四个小时能走10英里，时间= 4×60 分钟，速度为 $\frac{240}{10} = 24$ 分钟每英里。所以跟小时候比，走一英里差10分钟。

方法二：When Cheenu was a boy, he could run 15 miles in 3 hours and 30 minutes = $3 \times 60 + 30$ minutes = 210 minutes, thus running $\frac{210}{15} = 14$ minutes per mile.

When he is an old man, he can walk 10 miles in 4 hours = 4×60 minutes = 240 minutes, thus waking $\frac{240}{10} = 24$ minutes per mile. Therefore it takes him (B) 10 minutes longer to walk a mile now compared to when he was a boy.

5 (1分) The number N is a two-digit number.

- When N is divided by 9, the remainder is 1.
- When N is divided by 10, the remainder is 3.

What is the remainder when N is divided by 11? () .

- A. 0 B. 2 C. 4 D. 5 E. 7

Answer E

Solution 方法一：数字 N 是个两位数。

N 除以9，余数为1。

N 除以10，余数为3。

则 N 除以11时，余数为几？

从第二个子弹开始，我们知道第二个数字必须是**3**，因为除以**9**还有一个余数**1**，**9**的倍数必须以**2**结束，我们找到：

$$9(1) = 9$$

$$9(2) = 18$$

$$9(3) = 27$$

$$9(4) = 36$$

$$9(5) = 45$$

$$9(6) = 54$$

$$9(7) = 63$$

$$9(8) = 72$$

这个上数字 $71 + 1 = 73$ 都满足条件，我们减去**11**的小于**73**的最大倍数得到剩余的，也就是

$$73 - 11(6) = 73 - 66 = 7.$$

故选**E**。

方法二：From the second bullet point, we know that the second digit must be **3**.

Because there is a remainder of **1** when it is divided by **9** the multiple of **9** must end in a **2**. We now look for this one:

$$9(1) = 9$$

$$9(2) = 18$$

$$9(3) = 27$$

$$9(4) = 36$$

$$9(5) = 45$$

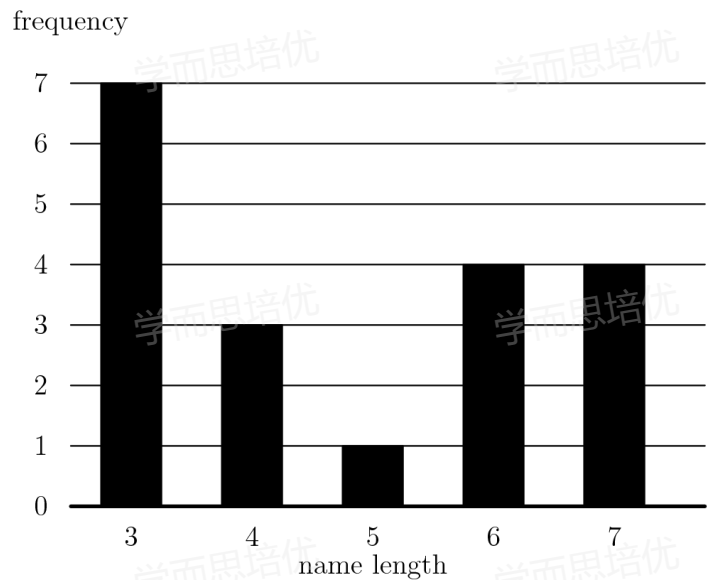
$$9(6) = 54$$

$$9(8) = 72$$

The number $72 + 1 = 73$ satisfies both conditions. We subtract the biggest multiple of

11 less than **73** to get the remainder. Thus, $73 - 11(6) = 73 - 66 = \boxed{\text{E}7}$.

- 6** (1分) The following bar graph represents the length (in letters) of the names of 19 people. What is the median length of these names? () .



A. 3

B. 4

C. 5

D. 6

E. 7

Answer B

Solution方法一：下图记录了19个人的名字长度（按字母个数计）。则这些长度的中位数是？

我们注意到中间的名字就是 10^{th} 的名字， 10^{th} 的名字是4。

故选B。

方法二：We first notice that the median name will be the 10^{th} name. The 10^{th} name is 4.

7 (1分) Which of the following numbers is not a perfect square? () .

A. 1^{2016}

B. 2^{2017}

C. 3^{2018}

D. 4^{2019}

E. 5^{2020}

Answer B

Solution方法一：下面哪个不是完全平方数？

我们知道，我们的答案必须有一个奇数指数，这样它才不是一个正方形。因为4是一个完全正方形， 4^{2019} 也是一个完全正方形，所以我们的答案一定是 2^{2017} 。

故选B。

方法二：We know that our answer must have an odd exponent in order for it to not be a square. Because 4 is a perfect square, 4^{2019} is also a perfect square, so our answer

must be 2^{2017} .

8 (1分) Find the value of the expression

$$100 - 98 + 96 - 94 + 92 - 90 + \cdots + 8 - 6 + 4 - 2. (\quad) .$$

A. 20

B. 40

C. 50

D. 80

E. 100

Answer C

Solution 方法一：计算该表达式的值。

我们可以将每对减法组合在一起：

$$(100 - 98) + (96 - 94) + (92 - 90) + \cdots + (8 - 6) + (4 - 2) .$$

$$\text{化简得 } 2 + 2 + 2 + \cdots + 2 + 2 = 2(1 + 1 + 1 + \cdots + 1 + 1) .$$

这里有50个偶数，所以有 $\frac{50}{2} = 25$ 对偶数，所以总和为 $2 \cdot 25 = 50$.

故选C .

方法二：因为我们的列表不以1结尾，所以我们将每个数字除以二，以

$50 - 49 + 48 - 47 + \cdots + 4 - 3 + 2 - 1$ 结尾。我们可以将每对减法组合在一起，

$(50 - 49) + (48 - 47) + (46 - 45) + \cdots + (4 - 3) + (2 - 1)$ 这有25对数字，且每个值为1，和

为25，然而，原来我们是除以2，所以再乘以2得到答案50 .

故选C .

方法三：We can group each subtracting pair together:

$$(100 - 98) + (96 - 94) + (92 - 90) + \cdots + (8 - 6) + (4 - 2) .$$

After subtracting, we have:

$$2 + 2 + 2 + \cdots + 2 + 2 = 2(1 + 1 + 1 + \cdots + 1 + 1) .$$

There are 50 even numbers, therefore there are $\frac{50}{2} = 25$ even pairs. Therefore the sum is $2 \cdot 25 = (C) 50$.

方法四：Since our list does not end with one, we divide every number by 2 and we

end up with $50 - 49 + 48 - 47 + \cdots + 4 - 3 + 2 - 1$. We can group each subtracting pair

together:

$(50 - 49) + (48 - 47) + (46 - 45) + \cdots + (4 - 3) + (2 - 1)$. There are now 25 pairs of numbers, and the value of each pair is 1. This sum is 25. However, we divided by 2 originally so we will multiply $2 * 25$ to get the final answer or 50.

9 (1分) What is the sum of the distinct prime integer divisors of 2016? () .

- A. 9 B. 12 C. 16 D. 49 E. 63

Answer B

Solution 方法一：2016的不同素因子之和是多少？

因式分解 $2016 = 2^5 \times 3^2 \times 7$ ，因为问题主要是找出不同的因数，那么它们的总和为12。

故选B。

方法二：The prime factorization is $2016 = 2^5 \times 3^2 \times 7$. Since the problem is only asking us for the distinct prime factors, we have 2, 3, 7. Their desired sum is then 12.

10 (1分) Suppose that $a * b$ means $3a - b$. What is the value of x if $2*(5*x) = 1$ () .

- A. $\frac{1}{10}$ B. 2 C. $\frac{10}{3}$ D. 10 E. 14

Answer D

Solution 方法一：若记号 $a*b$ 表示 $3a - b$ 。则 x 取值多少能使 $2*(5*x) = 1$ 。

让我们把 $(5 * x) = 1$ 插进 $3a - b$ ，那么有 $3(5) - x$ ，现在我们有 $2 * (15 - x) = 1$ ，把

$2 * (15 - x)$ 插进 $3a - b$ ，我们有 $6 - 15 + x = 1$ ， $-9 + x = 1$ ，解得 $x = 10$ 。

故选D。

方法二：Let us plug in $(5 * x) = 1$ into $3a - b$. Thus it would be $3(5) - x$. Now we have $2 * (15 - x) = 1$.

Plugging $2 * (15 - x)$ into $3a - b$, we have $6 - 15 + x = 1$. Solving for x we have

$$-9 + x = 1$$

$$x = \boxed{(D)10}.$$

11 (1分) Determine how many two-digit numbers satisfy the following property:

when the number is added to the number obtained by reversing its digits, the sum is 132. () .

- A. 5 B. 7 C. 9 D. 11 E. 12

Answer B

Solution 方法一：计算有多少两位数满足如下条件：将此数各个数位左右颠倒之后，与原数之和为132

我们可以用数字的形式写出两位数 $10a + b$ 、 $10a + b$ 的相反数是 $10b + a$ ，它们的总和是 $(10a + b) + (10b + a) = 132$ ， $11a + 11b = 132$ ， $a + b = 12$ ，我们可以找到数对 (a, b) ，使得 $a + b = 12$ ，因为 a 和 b 都是数字，它们都小于10，那么可以是 $(3, 9)$ 、 $(4, 8)$ 、 $(5, 7)$ 、 $(6, 6)$ 、 $(7, 5)$ 、 $(8, 4)$ 、 $(9, 3)$ ； $(3, 9)$ 、 $(4, 8)$ 、 $(5, 7)$ 、 $(6, 6)$ 、 $(7, 5)$ 、 $(8, 4)$ 、 $(9, 3)$ 。

故选B。

方法二：We can write the two digit number in the form of $10a + b$; reverse of $10a + b$ is $10b + a$. The sum of those numbers is: $(10a + b) + (10b + a) = 132$, $11a + 11b = 132$, $a + b = 12$. We can use brute force to find order pairs (a, b) such that $a + b = 12$. Since a and b are both digits, both a and b have to be integers less than 10. Thus our ordered pairs are $(3, 9)$; $(4, 8)$; $(5, 7)$; $(6, 6)$; $(7, 5)$; $(8, 4)$; $(9, 3)$ or $\boxed{(B)7}$ ordered pairs.

12 (1分) Jefferson Middle School has the same number of boys and girls. $\frac{3}{4}$ of the girls and $\frac{2}{3}$ of the boys went on a field trip. What fraction of the students on the field trip were girls? () .

- A. $\frac{1}{2}$ B. $\frac{9}{17}$ C. $\frac{7}{13}$ D. $\frac{2}{3}$ E. $\frac{14}{15}$

Answer B

Solution 方法一：某中学有相同数量的男生和女生。其中 $\frac{3}{4}$ 的女生和 $\frac{2}{3}$ 的男生参与了外出活动。试问学生中比例多少的人参与了此次活动？

将儿童的数量设为被2, 4和3整除的数字，在这个问题中，学校里的学生人数不是一个具体的数字，因为在这个问题中没有实际的数字，这样我们不需要计算出小数就可以计算出答案，

120是一个有效的数字，这有60个女孩和60个男孩，所以有 $60 \cdot \frac{2}{3} = 40$ 个男孩在旅途中，整个在旅途中的孩子的数量是85，所以女孩比例为 $\frac{45}{85}$ ，既 $\frac{9}{17}$ 。

故选B。

方法二：让 b 是男孩在学校的数量， g 是女孩在学校的数量，我们知道 $g = b$ ，那就意味着

$$\frac{3}{4}b + \frac{2}{3}b = \frac{17}{12}b \text{ 孩子在旅途中，其中 } \frac{3}{4}b \text{ 是女孩的数量，所以答案是 } \frac{\frac{3}{4}b}{\frac{17}{12}b} = \frac{9}{17}。$$

方法三：Set the number of children to a number that is divisible by two, four, and three. In this question, the number of children in the school is not a specific number because there are no actual numbers in the question, only ratios. This way, we can calculate the answer without dealing with decimals. 120 is a number that works. There will be 60 girls and 60 boys. So, there will be $60 \cdot \frac{3}{4} = 45$ girls on the trip and $60 \cdot \frac{2}{3} = 40$ boys on the trip. The total number of children on the trip is 85, so the fraction of girls on the trip is $\frac{45}{85}$ or $\frac{9}{17}$.

方法四：Let there be b boys and g girls in the school. We see $g = b$, which means

$$\frac{3}{4}b + \frac{2}{3}b = \frac{17}{12}b \text{ kids went on the trip and } \frac{3}{4}b \text{ kids are girls. So, the answer is } \frac{\frac{3}{4}b}{\frac{17}{12}b} = \frac{9}{17}, \text{ which is } \frac{9}{17}。$$

13 (1分) Two different numbers are randomly selected from the set $-2, -1, 0, 3, 4, 5$ and multiplied together. What is the probability that the product is 0? () .

- A. $\frac{1}{6}$ B. $\frac{1}{5}$ C. $\frac{1}{4}$ D. $\frac{1}{3}$ E. $\frac{1}{2}$

Answer D

Solution 方法一：从 $-2, -1, 0, 3, 4, 5$ 中任取不同两数乘起来。乘积为0的概率是多少？

只有其中一个数字是0的情况下这个数才是0，第二个数字有五种方法，这有 $\binom{6}{2}$ 种方法随意选择两个数字，是15，所以有 $\frac{5}{15} = \frac{1}{3}$ ，既 $\frac{1}{3}$ 。

故选D。

方法二：这里总共有30种可能，因为这些数字是不同的，我们想让0是这个数，那么其中一个数字是0，第一个数为0有五种可能，第二个数字选择0的方法有5种，所以找到 $\frac{1}{3}$ 。

故选D。

方法三：The product can only be 0 if one of the numbers is 0. Once we chose 0, there are 5 ways we can chose the second number, or $6 - 1$. There are $\binom{6}{2}$ ways we can

chose 2 numbers randomly, and that is 15. So, $\frac{5}{15} = \frac{1}{3}$ so the answer is $\boxed{(D) \frac{1}{3}}$.

方法四：There are a total of 30 possibilities, because the numbers are different. We want 0 to be the product so one of the numbers is . There are 5 possibilities where 0 is chosen for the first number and there are 5 ways for 0 to be chosen as the second number. We seek $\boxed{(D) \frac{1}{3}}$.

14 (1分) Karl's car uses a gallon of gas every 35 miles, and his gas tank holds 14 gallons when it is full. One day, Karl started with a full tank of gas, drove 350 miles, bought gallons of gas, and continued driving to his destination. When he arrived, his gas tank was half full. How many miles did Karl drive that day? () .

- A. 525 B. 560 C. 595 D. 665 E. 735

Answer A

Solution 方法一：Karl的车每35英里消耗1加仑汽油，且邮箱满载为14加仑。某天，他在油箱加满后开出350英里，买了8加仑汽油，并最终抵达目的地。当他到达时，油箱剩余一半。问那天他开了多远？

因为他每隔35公里就用一加仑汽油，在行驶350公里够用了 $\frac{350}{35} = 10$ 加仑汽油。所以还剩下 $14 - 10 = 4$ 加仑汽油，他买了8加仑汽油，所以加到12加仑。当他到达后，他有 $\frac{1}{2} \times 14 = 7$ 加仑的汽油，所以他在第二段旅途中用了5加仑汽油，所以第二段旅途中行驶了175公里，加上350公里，所以答案是 $350 + 175 = 525$ 公里。

故选A .

方法二：Since he uses a gallon of gas every 35 miles, he had used $\frac{350}{35} = 10$ gallons after 350 miles.

Therefore, after the first leg of his trip he had $14 - 10 = 4$ gallons of gas left. Then, he bought 8 gallons of gas, which brought him up to 12 gallons of gas in his gas tank. When he arrived, he had $\frac{1}{2} \cdot 14 = 7$ gallons of gas. So he used 5 gallons of gas on the second leg of his trip. Therefore, the second part of his trip covered $5 \cdot 35 = 175$ miles. Adding this to the 350 miles, we see that he drove $350 + 175 = \boxed{(A)525}$ miles.

15 (1分) What is the largest power of 2 that is a divisor of $13^4 - 11^4$? () .

- A. 8 B. 16 C. 32 D. 64 E. 128

Answer C

Solution 方法一： $13^4 - 11^4$ 的 2 的幂因子最大是多少？

首先我们对 $13^4 - 11^4 = (13^2)^2 - (11^2)^2$ 使用平方差法得到

$13^4 - 11^4 = (13^2 + 11^2)(13^2 - 11^2)$ ，再次利用平方差化简得

$(169 + 121)(13 + 11)(13 - 11) = 290 \cdot 24 \cdot 2 = (2 \cdot 8 \cdot 2) \cdot (3 \cdot 145)$ ，意识到我们不需要右边，因为它不含因子 2，所以答案为 32。

故选C .

方法二：First, we use difference of squares on $13^4 - 11^4 = (13^2)^2 - (11^2)^2$ to get

$13^4 - 11^4 = (13^2 + 11^2)(13^2 - 11^2)$. Using difference of squares again and simplifying, we get $(169 + 121)(13 + 11)(13 - 11) = 290 \cdot 24 \cdot 2 = (2 \cdot 8 \cdot 2) \cdot (3 \cdot 145)$. Realizing that we don't need the right-hand side because it doesn't contain any factor of 2, we see that the greatest power of 2 that is a divisor $13^4 - 11^4$ is 32.

16 (1分) Annie and Bonnie are running laps around a 400-meter oval track. They started together, but Annie has pulled ahead because she runs 25% faster than Bonnie. How many laps will Annie have run when she first passes Bonnie? () .

A. $1\frac{1}{4}$

B. $3\frac{1}{3}$

C. 4

D. 5

E. 25

Answer D

Solution 方法一：Annie和 Bonnie 沿400米椭圆形跑道跑圈。他们从同一位置（同一方向）开始，但 Annie 由于比 Bonnie 快所以领先。则当 Annie 第一次赶超 Bonnie 的时候 Annie 已经跑了多少圈？

邦尼每跑一圈，安妮跑四分之一圈，所以邦尼要跑四圈才能被追上。这意味着邦尼和安妮是平等的，安妮需要再跑一圈才能追上邦尼，也就是说安妮需要跑5圈。

故选D。

方法二：Each lap Bonnie runs, Annie runs another quarter lap, so Bonnie will run four laps before she is overtaken. This means that Annie and Bonnie are equal so that Annie needs to run another lap to overtake Bonnie. That means Annie will have run **(D)5** laps.

17 (1分) An ATM password at Fred's Bank is composed of four digits from 0 to 9, with repeated digits allowable. If no password may begin with the sequence 9, 1, 1 then how many passwords are possible? () .

A. 30

B. 72900

C. 9000

D. 9990

E. 9999

Answer D

Solution 方法一：Fred的银行 ATM 机密码由四位数构成，每位取自0到9，允许重复。若要求没有密码可以从9, 1, 1这个序列开始，则还有多少种可选的排列方式？

对于前三位数， $10^3 - 1 = 999$ 种组合，因为911是不允许的，对于最后一个数字，是可以为任意数字，所以答案是 $999 \cdot 10 = 9990$ 。

故选D。

方法二：算上不合适的情况我们找到10种，这是因为我们以9, 1, 1开始，最后一个数字是10个数字的任意一个，所以答案是 $10^4 - 10 = 9990$ 。

故选D。

方法三：For the first three digits, there are $10^3 - 1 = 999$ combinations since 911 is not allowed. For the final digit, any of the 10 numbers are allowed.

$$999 \cdot 10 = 9990 \rightarrow \boxed{(D)9990}.$$

方法四：Counting the prohibited cases, we find that there are 10 of them. This is because we start with 9, 1, 1 and we can have any of the 10 digits for the last digit. So our answer is $10^4 - 10 = \boxed{(D)9990}$.

18 (1分) In an All-Area track meet, 216 sprinters enter a 100-meter dash competition. The track has 6 lanes, so only 6 sprinters can compete at a time. At the end of each race, the five non-winners are eliminated, and the winner will compete again in a later race. How many races are needed to determine the champion sprinter? () .

- A. 36 B. 42 C. 43 D. 60 E. 72

Answer C

Solution 方法一：在全域田径赛中，216个短跑选手参与了100米项目比赛。跑道共6条赛道，是以每次只有六人同场竞技。每次赛跑过后，胜者继续参赛，其余五人遭淘汰。要决出冠军共需多少场比赛？

从 n 种中，只有 $\frac{1}{6}$ 会继续，因为我们想知道比赛的总次数，一个随着时间而发生的比赛的列是一个想法，因为我们希望找到比赛的总数： $\frac{216}{6} = 36$ 、 $\frac{36}{6} = 6$ 、 $\frac{6}{6} = 1$ ，将第二列的所有数字相加得到43。

故选C。

方法二：每场比赛淘汰5人，胜利者是最后一个跑步者决定的。所以，有215人不得不淘汰，因此我们需要 $\frac{215}{5}$ 场比赛去决定谁是胜利者。

故选C。

方法三：From any n -th race, only $\frac{1}{6}$ will continue on. Since we wish to find the total number of races, a column representing the races over time is dealt Starting with the first race: $\frac{216}{6} = 36$ $\frac{36}{6} = 6$ $\frac{6}{6} = 1$ Adding all of the numbers in the second column

yields (C)43.

方法四：Every race eliminates 5 players. The winner is decided when there is only 1 runner left.

Thus, 215 players have to be eliminated. Therefore, we need $\frac{215}{5}$ games to decide the winner, or (C)43.

19 (1分) The sum of 25 consecutive even integers is 10,000. What is the largest of these 25 consecutive integers? () .

- A. 360 B. 388 C. 412 D. 416 E. 424

Answer E

Solution 方法一：25个连续偶数的和是10000。这25个数中最大的一个是多少？

设 n 是被加起来的第13个连续偶数整数。通过这样作，我们可以看到25个偶数的和可以化简为

$25n$ ，因为 $(n-2k) + \dots + (n-4) + (n-2) + (n) + (n+2) + (n+4) +$

$\dots + (n+2k) = 25n$ ，

现在， $25n = 10000 \rightarrow n = 400$ ，记住这是第13个整数，我们想找到第25个是

$400 + 2(25 - 13) = 424$ 。

故选E。

方法二：Let n be the 13th consecutive even integer that's being added up. By doing this, we can see that the sum of all 25 even numbers will simplify to $25n$ since

$(n-2k) + \dots + (n-4) + (n-2) + (n) + (n+2) + (n+4) + \dots + (n+2k) = 25n$

Now, $25n = 10000 \rightarrow n = 400$ Remembering that this is the 13th integer, we wish to

find the 25th, which is $400 + 2(25 - 13) = \boxed{(E)424}$.

20 (1分) The least common multiple of a and b is 12, and the least common multiple of b and c is 15. What is the least possible value of the least common multiple of a and c ? () .

- A. 20 B. 30 C. 60 D. 120 E. 180

Answer A

Solution 方法一： a 和 b 的最小公倍数是12， b 和 c 的最小公倍数是15。则 a 和 c 的最小公倍数是最小可能是？

我们希望能找到 a ， b 和 c 的值，通过找到12和15的最大公因数，从代数的角度来看，它是 b 的倍数，从数字来看我们确实是3，所以 b 为3，转移到 a 和 c ，我们为了求他们的最小值，我们希望找到最小值， a 和3的公倍数是12，所以 a 为4对3和 c ，类似地得到 c 为5，那么4和5的最小公倍数是20。

故选A。

方法二：We wish to find possible values of a , b , and c . By finding the greatest common factor of 12 and 15, algebraically, it's some multiple of b and from looking at the numbers, we are sure that it is 3, thus b is 3. Moving on to a and c , in order to minimize them, we wish to find the least such that the least common multiple of a and 3 is 12, $\rightarrow 4$. Similarly with 3 and c , we obtain 5. The least common multiple of 4 and 5 is 20 $\rightarrow$ **(A)20**.

21 (1分) A top hat contains 3 red chips and 2 green chips. Chips are drawn randomly, one at a time without replacement, until all 3 of the reds are drawn or until both green chips are drawn. What is the probability that the 3 reds are drawn? () .

A. $\frac{3}{10}$

B. $\frac{2}{5}$

C. $\frac{1}{2}$

D. $\frac{3}{5}$

E. $\frac{7}{10}$

Answer B

Solution 方法一：一顶高礼帽中有3块红色卡片和2块绿色卡片。卡片每次不放回地随机抽取一张，直到某一个颜色的卡片被全部抽出停止。则3张红色卡片被抽出的概率是？

我们随机地排列了五个芯片，然后从左到右挑选芯片，为了找到三个红色筹码和两个绿色芯片的排列方法，我们计算 $\binom{5}{2} = 10$ ，然而我们注意到，我们在画最后一个红色芯片时，我们先选择两个绿色芯片，再选择最后一个红色芯片。同样的，当最后一个芯片是绿色时，在最

后一个绿色芯片之前，我们都选择三个红色芯片，这意味着在所有可行的情况下，最后一个芯片必须是绿色，这意味着我们只能找到重新排列的三个红色芯片和一个绿色芯片，它是 $\binom{4}{3} = 4$ ，因为一个绿色芯片至少是10种情况中的四种，我们的答案是 $\frac{2}{5}$ 。

故选B。

方法二：有两种结束游戏的方法，要么你挑出所有的红色芯片，要么你挑出所有绿色芯片，我们可以挑出三个红色芯片，三个红色芯片和一个绿色芯片，两个绿色芯片，两个绿色芯片和一个红色芯片，两个绿色芯片和两个红色芯片。因为顺序在这个问题上很重要，这有 $1 + 4 + 1 + 3 + 6 = 15$ 种方法去挑选芯片，但是我们注意到在挑出一个绿色芯片之前先挑出三个红色芯片，这个游戏结束，所以我们需要减去这种情况的情况才能找到比赛结束的方法数，是 $15 - 5 = 10$ 。在10种游戏结束的方法中，有四种是以红色芯片结束的。答案是 $\frac{4}{10} = \frac{2}{5}$ 。

故选B。

方法三：We put five chips randomly in order, and then pick the chips from the left to the right. To find the number of ways to rearrange the three red chips and two green chips, we solve for $\binom{5}{2} = 10$. However, we notice that whenever the last chip we draw is red, we pick both green chips before we pick the last (red) chip. Similarly, when the last chip is green, we pick all three red chips before the last (green) chip. This means that the last chip must be green in all the situations that work. This means we are left with finding the number of ways to rearrange three red chips and one green chip, which is $\binom{4}{3} = 4$. Because a green chip will be last 4 out of the 10 situations, our answer is $\frac{4}{10} = \frac{2}{5}$.

方法四：There are two ways of ending the game, either you picked out all the red chips or you picked out all the green chips. We can pick out 3 red chips, 3 red chips and 1 green chip, 2 green chips, 2 green chips and 1 red chip, and 2 green chips and 2 red chips. Because order is important in this problem, there are

$1 + 4 + 1 + 3 + 6 = 15$ ways to pick out the chip. But we noticed that if you pick out the three red chips before you pick out the green chip, the game ends. So we need to subtract cases like that to get the total number of ways a game could end, which $15 - 5 = 10$. Out of the 10 ways to end the game, 4 of them ends with a red chip. The

answer is $\frac{4}{10} = \frac{2}{5}$, or $\boxed{\text{(B)} \frac{2}{5}}$.

- 22 (1分) Rectangle $DEFA$ below is a 3×4 rectangle with $DC = CB = BA$. What is the area of the "bat wings" (shaded area)? () .

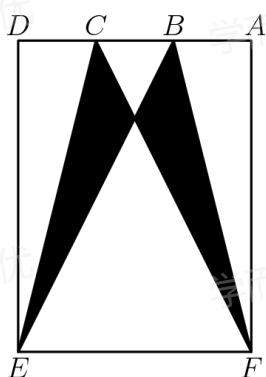


- A. 2 B. $2\frac{1}{2}$ C. 3 D. $3\frac{1}{2}$

E. 4

Answer C

Solution 方法一：如下图的 3×4 矩形 $DEFA$ ，满足 $DC = CB = BA$ ，则图中蝙蝠图案（阴影区域）面积是？

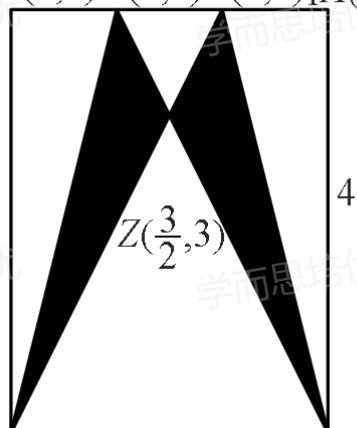


梯形 $CBFE$ 的面积是 $\frac{1+3}{2} \times 4 = 8$ ，接下来，我们找到每个三角形的高去计算它们的面积，这些三角形是相似的，相似比为 $3:1$ ，所以每个高是 3 ，而较小的那个高是 1 。因此，它们的面积是 $\frac{1}{2}$ 和 $\frac{9}{2}$ 。从梯形中减去这些面积我们得到 $8 - \frac{1}{2} - \frac{9}{2} = 3$ 。

故选 C。

方法二：设写坐标，令 $E = (0, 0)$ ， $F = (3, 0)$ ，

$D(0,4)C(1,4)B(2,4)A(3,4)$



$E(0,0)$ $F(3,0)$

现在，我们容易发现线 CF 的格点坐标为 $(1,4)$ 和 $(3,0)$ 。因此，直线的斜率为 $CF = -2$ ，插入其他点的坐标。我们发现线 $CF = -2x + 6$ ，对线 BE 进行同样的操作。我们得到 $BE = 2x$ 。

因此，建立方程求出交点。 $y = 2x = -2x + 6 \Rightarrow 4x = 6 \Rightarrow x = \frac{3}{2} \Rightarrow y = 3$ ，那么交点为 $\left(\frac{3}{2}, 3\right)$ 称为 Z ，现在我们知道 $E = (0,0)$ 、 $Z = \left(\frac{3}{2}, 3\right)$ 、 $C = (1,4)$ 。

使用著名的鞋带公式，这些阴影小三角形之一的面积为 $\frac{3}{2}$ ，因为其中有两个，所以乘以二得到答案3。

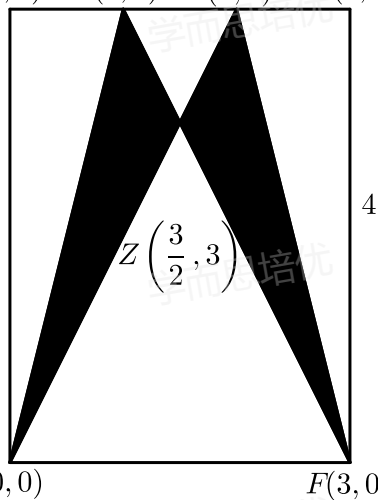
故选C。

方法三：The area of trapezoid $CBFE$ is $\frac{1+3}{2} \cdot 4 = 8$. Next, we find the height of each triangle to calculate their area. The triangles are similar, and are in a $3:1$ ratio by AA similarity (alternate interior and vertical angles) so the height of the larger one is 3, while the height of the smaller one is 1. Thus, their areas are $\frac{1}{2}$ and $\frac{9}{2}$. Subtracting these areas from the trapezoid, we get $8 - \frac{1}{2} - \frac{9}{2} = \boxed{3}$. therefore, the answer to this problem is $\boxed{(C)3}$.

方法四：Setting coordinates!

Let $E = (0,0)$, $F = (3,0)$

$D(0, 4)$ $C(1, 4)$ $B(2, 4)$ $A(3, 4)$



Now, we easily discover that line CF has lattice coordinates at $(1, 4)$ and $(3, 0)$.

Hence, the slope of line $CF = -2$

Plugging in the rest of the coordinate points, we find that line $CF = -2x + 6$

Doing the same process to line BE , we find that line $BE = 2x$

Hence, setting them equal to find the intersection point...

$$y = 2x = -2x + 6 \Rightarrow 4x = 6 \Rightarrow x = \frac{3}{2} \Rightarrow y = 3.$$

Hence, we find that the intersection point is $\left(\frac{3}{2}, 3\right)$. Call it Z .

Now, we can see that

$$E = (0, 0)$$

$$Z = \left(\frac{3}{2}, 3\right)$$

$$C = (1, 4).$$

Shoelace!

Using the well known Shoelace Formula, we find that the area of one of those small shaded triangles is $\frac{3}{2}$.

Now because there are two of them, we multiple that area by 2 to get $\boxed{(C)3}$.

23

(1分) Two congruent circles centered at points A and B each pass through the other circle's center. The line containing both A and B is extended to intersect the circles at points C and D . The circles intersect at two points, one of which is E . What is the degree measure of $\angle CED$? () .

A. 90

B. 105

C. 120

D. 135

E. 150

Answer C

Solution 方法一：圆心在 A 和 B 的两等圆，各自经过另一点。过 A 和 B 的直线另交两圆于 C 和 D 。两圆交点中称其一为 E 。则 $\angle CED$ 的角度是？

我们知道 $\triangle EAB$ 是等边三角形，因为每边都是两个圆的半径。因此， $\widehat{EB} = \angle EAB = 60^\circ$ ，所以这是一个内切角。

故选 C。

方法二：因为在第一种解法中得到 $\triangle EAB$ 是等边三角形。所以，

$m\angle AEB = m\angle EAB = m\angle EBA = 60^\circ$ ，因为 CD 是一条直线，我们能够推断出

$m\angle EBD = 180^\circ - 60^\circ = 120^\circ$ ，因为 $BE = BD$ （两者都是同一圆的半径）， $\triangle BED$ 是等腰三角形，意味着 $m\angle BED = m\angle BDE = 30^\circ$ ，同样地， $m\angle AEC = m\angle ACE = 30^\circ$ 。现在， $\angle CED = m\angle AEC + m\angle AEB + m\angle BED = 30^\circ + 60^\circ + 30^\circ = 120^\circ$ 。

故选 C。

方法三：Drawing the diagram (lost):

we see that $\triangle EAB$ is equilateral as each side is the radius of one of the two circles. Therefore, $\widehat{EB} = m\angle EAB = 60^\circ$. Therefore, since it is an inscribed angle

方法四：As in Solution 1, observe that $\triangle EAB$ is equilateral.

Therefore, $m\angle AEB = m\angle EAB = m\angle EBA = 60^\circ$.

Since CD is a straight line,

we conclude that $m\angle EBD = 180^\circ - 60^\circ = 120^\circ$. Since $BE = BD$ (both are radi of the same circle), $\triangle BED$ is isosceles, meaning that $m\angle BED = m\angle BDE = 30^\circ$.

Similarly, $m\angle AEC = m\angle ACE = 30^\circ$.

Now, $\angle CED = m\angle AEC + m\angle AEB + m\angle BED = 30^\circ + 60^\circ + 30^\circ = 120^\circ$ Therefore, the answer is 120.

24 (1分) The digits 1, 2, 3, 4, and 5 are each used once to write a five-digit number $PQRST$. The three-digit number PQR is divisible by 4, the three-digit number QRS

is divisible by 5, and the three-digit number RST is divisible by 3. What is P ? () .

- A. 1 B. 2 C. 3 D. 4 E. 5

Answer A

Solution 方法一：1, 2, 3, 4, 5各用一次组成五位数 $PQRST$ 。若三位数 PQR 被4整除， QRS 被5整除，且 RST 被3整除，则的值是？

我们知道 QRS 可被5整除， S 一定是0或者5，但是它不能等于0，所以 $S = 5$ ，我们注意到 PQR 一定是偶数， R 一定为2或者4，然而 $R = 2$ ，我们知道 $T \equiv 2 \pmod{3}$ ，这种情况不能发生，因为2和5已经用过了，所以 $R = 4$ ，这给了 $T \equiv 3 \pmod{4}$ ，意味着 $T = 3$ ，现在我们知道 Q 为1或者2，但是14不能被4整除，但是24可以，所以 $R = 4$ ，并且 $P = 1$ 。

故选A。

方法二：我们知道 $PQRST$ 、 ORS 是被5整除，因此 S 明显是5。因为 QRS 是被5整除的，所以我们有 $PQR5T$ 是我们的数量，接下来，让我们继续了解提供的第二个信息， RST 被3整除。所以，根据这一点 RST 的总和是3的倍数，只有两个足够大的是9和12，因为5已经给出， RT 的总和可能是4和7，所以 T 的值可能是1, 3, 4, 3, T 的值可能是3、1、3、4。所以利用这个我们可以继续讨论这个事实， PQR 是被4整除的，所以利用这个我们知道 R 是偶数，所以它的值只有可能是4，我们也知道 R 的值也只可能为3。所以，我们有 $PQRST = PQ453$ ，所以 P 和 Q 的值可能为1或者2，我们知道 QR 能被4整除，

所以可能为14或者24，24能够被4整除，所以 P 只能是1，答案为 $P = 1$ 。

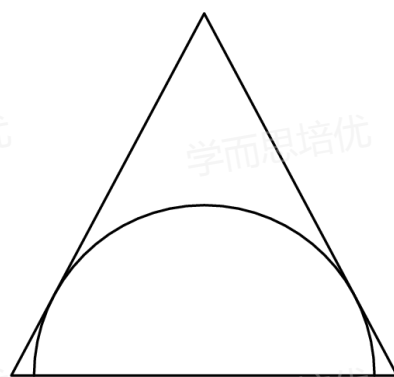
故选A。

方法三：We see that since QRS is divisible by 5, S must equal either 0 or 5, but it cannot equal 0, so $S = 5$. We notice that since PQR must be even, R must be either 2 or 4. However, when $R = 2$, we see that $T \equiv 2 \pmod{3}$, which cannot happen because 2 and 5 are already used up; so $R = 4$. This gives $T \equiv 3 \pmod{4}$, meaning $T = 3$. Now. We see that Q could be either 1 or 2. But 14 is not divisible by 4, but 24 is. This means that $R = 4$ and $P = \boxed{(A)1}$.

方法四：We know that out of $PQRST$, QRS is divisible by 5. Therefore S is obviously 5 because QRS is divisible by 5. So we now have $PQR5T$ as our number. Next, lets

move on to the second piece of information that was given to us. RST is divisible by 3. So, according to the divisibility of 3 rule the sum of RST has to be a multiple of 3. The only 2 big enough is 9 and 12 and since 5 is already given. The possible sums of RT is 4 and 7. So, the possible values for R are 1, 3, 4, 3 and the possible values of T is 3, 1, 3, 4. So, using this we can move on to the fact that PQR is divisible by 4. So, using that we know that R has to be even so 4 is the only possible value for R . Using that we also know that 3 is the only possible value for 3. So, we know have $PQRST = PQ453$ so the possible values are 1 and 2 for P and Q . Using the divisibility rule of 4 we know that QR has to be divisible by 4. So, either 14 or 24 are the possibilities, and 24 is divisible by 4. So the only value left for P is 1. $P = \boxed{(A)1}$.

- 25 (1分) A semicircle is inscribed in an isosceles triangle with base 16 and height 15 so that the diameter of the semicircle is contained in the base of the triangle as shown. What is the radius of the semicircle? () .



A. $4\sqrt{3}$

B. $\frac{120}{17}$

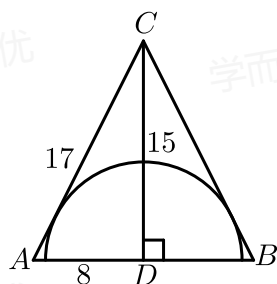
C. 10

D. $\frac{17\sqrt{2}}{2}$

E. $\frac{17\sqrt{3}}{2}$

Answer B

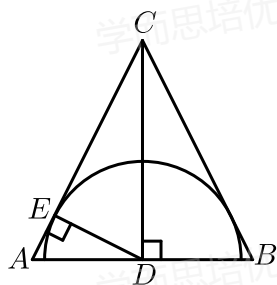
Solution 方法一：一半圆内切于一个底16，高15的等腰三角形，使该半圆直径与三角形的底共线。试问半圆半径。



首先，我们画一条垂直于三角形底部的线，把三角形分成两半。得到的直角三角形的底部是8，高是15，利用勾股定理有斜边的长度为17，用直角三角形的两个直角边我们能得到三角形的面积为60，经过 $\frac{60}{17}$ 次半径增大2，这是当斜边为基地时右三角形的高。

故选B。

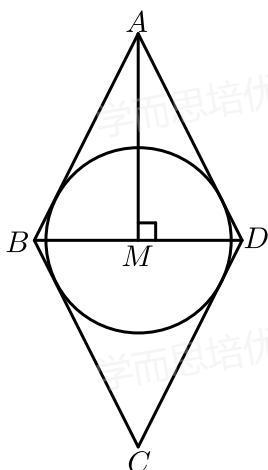
方法二：



在三角形 $\triangle ABC$ ，其中 $AB = 16$ 和 $AC = BC$ 。D是AB的中点，E是边AC与圆相切的点，因为对称，我们也利用BC代替AC，注意到 $\triangle ACD \cong \triangle BCD$ ，都是8-15-17直角三角形。我们还知道，我们作出直线与半径的交点和圆的切线。所以通过相似 $\triangle AED \sim \triangle ADC$ ，有 $\angle EAD \cong \angle DAC$ 和 $\angle CDA \cong \angle DEA$ ，由相似我们得出 $\frac{AD}{AC} = \frac{DE}{CD}$ ， $AD = \frac{AB}{2} = 8$ ， $AC = 17$ 和 $CD = 15$ ，两边乘以15得 $DE = \frac{8}{17} \times 15 = \frac{120}{17}$ 。

故选B。

方法三：



我们称三角形为 $\triangle ABD$ ，令BD的中点为M，过A点作直线AM平分直线BD， $\triangle ABD$ 的底的一半为 $\frac{16}{2} = 8$ ，高为15。利用勾股定理AB和DA的长为17，在底座上反射三角形，这将在菱形ABCD中创建一个内切圆。因为 $AB \cong DA$ ， $BC \cong CD$ ，所以 $AB = BC = CD = DA$ 。菱形

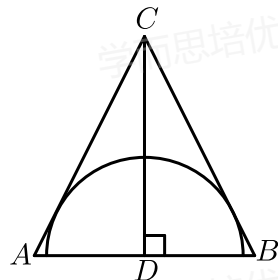
的半周长是 $\frac{AB + BC + CD + DA}{2} = \frac{(17)(4)}{2} = 34$. 因为 $\triangle ABD$ 的面积为 $\frac{bh}{2}$, 是菱形面积的两倍, 所以菱形面积为 $bh = (16)(15) = 240$. 把半周长和面积写成方程得 $r = \frac{240}{34} = \frac{120}{17}$.

故选B .

方法四：现在我们有 $8 - 15 - 17$ 直角三角形, 我们能找到 AE 和 CE , 令 $AE = x$, $CE = 17 - x$, 那么通过相似三角形我们有 $15^2 = 17x$, 所以 $AE = x = \frac{225}{17}$, $CE = \frac{64}{17}$. 现在被称之为“斜边的高度” , $r = \sqrt{AE \times CE}$, 所以 $r = \frac{120}{17}$.

故选B .

方法五：



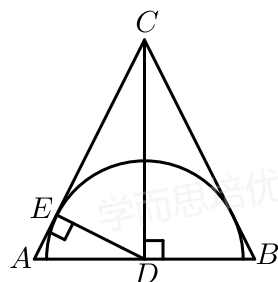
注意：这个上问题使用三角形概念 .

A 表示等腰三角形的左下顶点, B 表示等腰三角形的右下顶点. C 表示三角形的顶点. 过 C 点向 AB 边作高, 交 BA 于点 D . 通过勾股定理我们得 $AC = 17$.

现在我们知道 $\sin A = \frac{15}{17}$, 又因为 $\sin A = \frac{r}{8}$, r 为半圆半径, 所以 $r = \frac{120}{17}$.

故选B .

方法六：



这里有两种方法求出 $\triangle ABC$ 的面积, 第一种方法最显然, 在某种程度上更复杂一些, 现在 $\triangle ACD$ 和 $\triangle BCD$ 是全等的, 这意味着只要算出其中一个三角形面积, 那么乘以2就是 ABC 的面积, 但是因为我们总是分为两个, 在寻求三角形的面积公式中, 乘以二除以二抵消 .

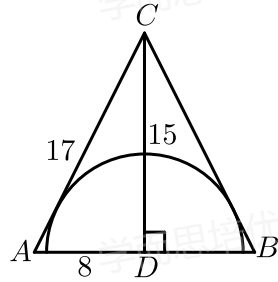
如果我们只是将 $\triangle ACD$ 底和高相等, 我们将得到 $\triangle ABC$ 的面积, 现在你可能认为: “找到面积的另一方法是什么” , 底 AC 很容易知道, 高是 DE , 也是半径, 第一种方法给出了

$\triangle ABC$ 的面积 $\frac{15 \times 16}{2} = 120$, 那么 $120 = AC \cdot r$, r 为半径, 在 $\triangle ACD$ 中利用勾股定理我们可以求出 AC , 两个直角边为8和15, 所以 AC 为17, 现在有一个半径和一个方程,

$120 = 17r$, 我们解出半径得 $r = \frac{120}{17}$.

故选B .

方法七：

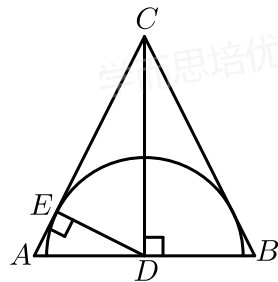


First, we draw a line perpendicular to the base of the triangle and cut the triangle in half. The base of the resulting right triangle would be 8, and the height would be 15.

Using the Pythagorean theorem, we can find the length of the hypotenuse, which would be 17. Using the two legs of the right angle, we can find the area of the right triangle, 60 . $\frac{60}{17}$ times 2 results in the radius, which is the height of the right triangle

when using the hypotenuse as the base. The answer is $\boxed{(B) \frac{120}{17}}$.

方法八：



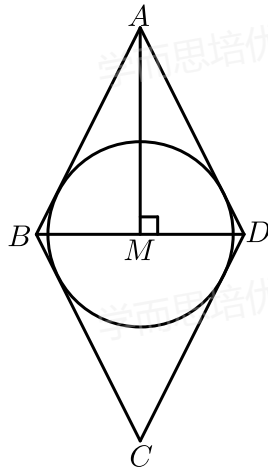
Let's call the triangle $\triangle ABC$, where $AB = 16$ and $AC = BC$. Let's say that D is the midpoint of AB and E is the point where AC is tangent to the semicircle. We could also use BC instead of AC because of symmetry.

Notice that $\triangle ACD \cong \triangle BCD$, and are both $8 - 15 - 17$ right triangles. We also know that we create a right angle with the intersection of the radius and a tangent line of a circle (or part of a circle). So, by AA similarity, $\triangle AED \sim \triangle ADC$, with $\angle EAD \cong \angle DAC$ and $\angle CDA \cong \angle DEA$. This similarity means that we can create a proportion:

$\frac{AD}{AC} = \frac{DE}{CD}$. We plug in $AD = \frac{AB}{2} = 8$, $AC = 17$, and $CD = 15$. After we multiply both sides by 15, we get $DE = \frac{8}{17} \cdot 15 = \boxed{(B) \frac{120}{17}}$.

(By the way, we could also use $\triangle DEC \sim \triangle ADC$.)

方法九：



We'll call this triangle $\triangle ABD$. Let the midpoint of base BD be M . Divide the triangle in half by drawing a line from A to M . Half the base of $\triangle ABD$ is $\frac{16}{2} = 8$. The height is 15, which is given in the question. Using the Pythagorean Triple 8 – 15 – 17, the length of each of the legs (AB and DA) is 17.

Reflect the triangle over its base. This will create an inscribed circle in a rhombus $ABCD$.

Because $AB \cong DA$, $BC \cong CD$. Therefore $AB = BC = CD = DA$.

The semiperimeter s of the rhombus is $\frac{AB + BC + CD + DA}{2} = \frac{(17)(4)}{2} = 34$. Since the area of $\triangle ABD$ is $\frac{bh}{2}$, the area $[ABCD]$ of the rhombus is twice that, which is $bh = (16)(15) = 240$.

The Formula for the Incircle of a Quadrilateral is $sr = [ABCD]$. Substituting the

semiperimeter and area into the equation, $34r = 240$. Solving this, $r = \frac{240}{34} = \boxed{(B) \frac{120}{17}}$

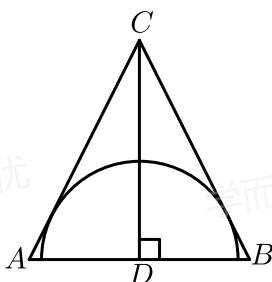
方法十：Noting that we have a 8 – 15 – 17 triangle, we can find AE and CE . Let

$AE = x$, $CE = 17 - x$. Then by similar triangles (or "Altitude on Hypotenuse") we have

$15^2 = x * 17$. Thus, $AE = x = \frac{225}{17}$, $CE = \frac{64}{17}$. Now again by "Altitude on Hypotenuse",

$r = \sqrt{AE * CE}$. Therefore $r = \frac{120}{17}$.

方法十一：Note: This solution uses Trigonometric Concepts



Denote the bottom left vertex of the isosceles triangle to be A

Denote the bottom right vertex of the isosceles triangle to be B

Denote the top vertex of the isosceles triangle to be C

Drop an altitude from C to side AB . Denote the foot of intersection to be D .

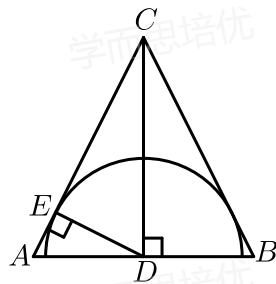
By the Pythagorean Theorem, $AC = 17$.

Now, we see that $\sin A = \frac{15}{17}$.

This implies that $\sin A = \frac{r}{8}$ (r = radius of semicircle).

Hence, $r = \boxed{\frac{120}{17}}$.

方法十二：



Credits for Asymptote go to whoever wrote this diagram up in Solution 2. There are two ways to find the area of $\triangle ABC$. The first way is the most obvious, and that is to multiply the base times the height ($16 \cdot 15$) and then divide it by two. The second way is, in a way, a little more complex. Note that $\triangle ACD$ and $\triangle BCD$ are congruent. This means that if we find the area of one triangle, we can just multiply it's area by 2 and we've found the area of the larger triangle ABC. But since we always divide by two, as it is in the formula for finding the area of a triangle, the multiply by two and divide by two cancel out, giving us that if we just multiply base times height of $\triangle ACD$, we will get the area of $\triangle ABC$. Now you might think: 'But what is the other way of finding the area'. Well that is AC , which would be the base, times DE , the radius, which would be the height. The first way to find the area gives us the area of the $\triangle ABC$, $\frac{15 \cdot 16}{2} = 120$. This gives us $120 = AC \cdot r$ (r signifies radius). We can find AC using the Pythagorean Theorem on $\triangle ACD$. The two legs are 8 and 15, which gives us that the hypotenuse, AC , is equal to 17. Now that we have an equation with only one variable for the radius, $120 = 17r$, we can just solve for the radius. We get $r = \frac{120}{17}$. This may seem like a lengthy explanation, but doing it yourself when you know what to do, it actually takes very little time. Try it yourself!

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优

学而思培优