



The Ultimate Geometry Guide for Middle Schoolers

Featuring Past Math Contest Questions &
In-Depth Explanations

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5

Contents

Lesson 1 Angles 1

Lesson 2 Find the Length 10

Lesson 3 Area and Perimeter 19

Lesson 4 Volume 27



 <https://www.thethinkacademy.com/>

Think Academy US ONLINE

Middle School Math Program

Think Academy's Middle School Long-Term Math Program is designed for Grades 6–9, offering a professional, systematic, and comprehensive year-round curriculum tailored to North American students. The program includes four tracks: Core+, Honors, Challenge, and Competition.

Core+ (School Curriculum Track): Focuses on previewing and reinforcing key concepts through targeted practice, helping students easily achieve A grades in school math at their current grade level.

Honors (Accelerated Advancement Track): One year ahead of standard pace, matching the fastest public-school classes. It deepens understanding and aims for placement in advanced tracks, completion of 5 AP STEM courses before graduation, and perfect SAT/ACT Math scores.

Challenge: Two years ahead of standard pace, covering Algebra 1, Geometry, and Algebra 2 in two years. Align with top private schools, preparing students to complete 8 AP STEM courses and get perfect SAT/ACT Math results by graduation.

Competition: Design around the AMC syllabus, build competitive math skills through advanced, contest-level topics. Taught by an experienced coaching team with a proven record of training AMC and AIME winners, it helps students qualify for AIME by Grades 8–9.

	Year 1			Year 2			Year 3			Year 4			Year 5		
	Summer	Fall	Spring	Summer	Fall	Spring	Summer	Fall	Spring	Summer	Fall	Spring	Summer	Fall	Spring
Core+	Math 6/7a			Math 7b/8			Algebra 1			Geometry			Algebra 2		
Honors	Pre-Algebra			Algebra 1			Geometry			Algebra 2/ Trigonometry			Pre-Calculus		
Challenge	Pre-Algebra+			Algebra 1			Intro to Geo			Geometry			Algebra 2		
Competition															
				AMC 8 HR			AMC 10 Introduction			AMC 10 AIME qualify			AMC 12+ AIME		

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Middle School Competition Track

Student Results

2022-2024 AMC8 Number of Students Awarded:



Achievement Roll
(Lower Grade
Achievement Award)



Honor Roll
(National Top 5%)



DHR
(National Top 1%)

2022-2024 AMC10 Number of Students Awarded:



AIME Qualification
(National Top 7%)



Honor Roll
(National Top 5%)



DHR
(National Top 1%)

2024 Think Global IMO Award Winner

7金 1银

金牌总数居全球第一



International Mathematical Olympiad
Foundation Bronze Sponsor

Think Academy is an official partner of the
International Mathematical Olympiad (IMO)

Why Think Academy Competition Courses Trained Thousands of Award-Winning Students?

1 One-Stop Professional System for Math Competitions

Think Competition follows the AMC syllabus and aligns with students' learning pace. **Grades 5–6 focus on AMC 8**, while **Grades 7–8 focus on AMC 10**. The curriculum covers the four core modules of math competitions — Algebra, Geometry, Number Theory, and Counting & Probability — enabling students to achieve one competition milestone each year and **complete AMC 10 preparation before high school, successfully qualifying for AIME**.

2 Expert Coaches. Top Results. Proven Success.

Think Competition courses are taught by **instructors with many years of competition teaching experience**, enabling students to master complex contest concepts efficiently and with ease.



James
Upenn - 5 YOE



Dennis
Duke - 4 YOE



Yichen
Columbia - 4 YOE

3 Collaborate and Excel with Outstanding Peers

Each year, the Competition Track begins with an entrance assessment to ensure students learn alongside peers of similar ability. This structure makes preparation more engaging and collaborative, as students motivate one another and grow together throughout the journey.

Course Highlights

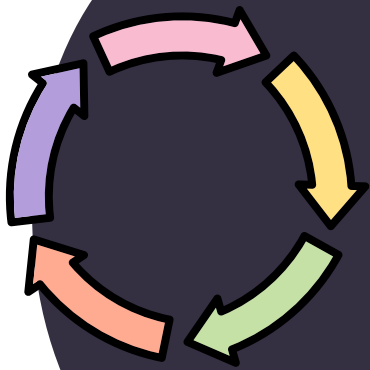
Clear Structure, Worry-Free for Parents

Personalized Learning Plans with Full Progress Tracking

- **Before Enrollment:** Teachers create **personalized study plans** based on each student's goals and ability
- **During Learning:** Teachers give **regular feedback and targeted advice**, helping parents track progress
- **Mid-Term & Final Evaluations:** **Parent meetings** and updated study plans ensure steady improvement



Weekly Learning Cycle to Ensure Steady Progress



- **Pre-Class Preparation:** **15-minute preview** to review key concepts and prepare for new lessons
- **Homework:** **Assigned after each class**, with teacher feedback shared with parents
- **Office Hour:** **Free live sessions** to review and clarify difficult problems
- **Homework Review Videos:** Each homework question includes a **detailed video explanation** for easy review

Year-Round Learning Support

- **Year-round Parent App service** providing fast responses
- Through the Parent App, parents get **direct contact with teachers** for efficient communication and real-time progress updates
- Instant online homework help for **timely learning support**

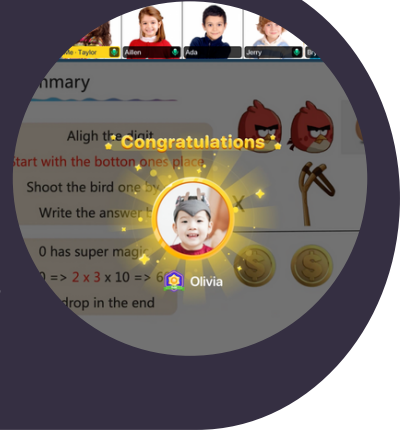


Course Highlights

Happy Learning, Better Results

Thoughtfully Crafted Lessons Inspire a Love for Math

- **Interactive games and scenario-based teaching** make math fun and engaging
- **Gold coin reward system** lets students exchange coins for prizes, motivating them to complete learning tasks actively



Self-Developed Learning App Enhances Every Online Class

- **Think Academy learning app** enhances in-class interaction with hand-raising, polls, and group activities —**keeping students engaged every three minutes**
- Teachers **track progress in real time** for an effective learning experience



A Sense of Achievement Builds Intrinsic Motivation

- Students learn efficiently in class and get **full after-class support** with detailed homework videos and free weekly Office Hours.
- Students **gain higher grades, faster progress**. They are naturally motivated to put more effort in STEM learning and become **confident math learners**



Lesson 1 Angles



Concept 1: Classification and Notation of Angles

1. Angle: An **angle** is composed of two different rays with the same endpoint.

2. Classification of Angles:

(1) A **straight angle** is an angle whose measure is 180° .

(2) A **right angle** is an angle whose measure is 90° .

(3) An acute angle is an angle whose measure is greater than 0° and less than 90° .

(4) An obtuse angle is an angle whose measure is greater than 90° and less than 180° .

3. **Notation of an angle:** An angle in geometry is denoted by the **angle symbol** ($\angle$) followed by **three letters** which represent points that form the angle.

4. Complementary and supplementary angles:

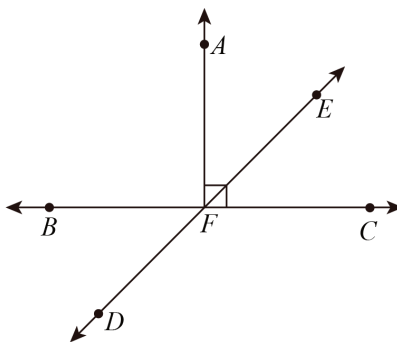
Two angles are **complementary angles** if sum of their measures is 90° ; two angles are **supplementary angles** if if sum of their measures is 180° .

5. Vertical Angles and Adjacent angles

Vertical angles: Two opposite angles which are formed by two intersecting lines. They have the same measures.

Adjacent angles: Two angles that share the same vertex and exactly one side. The sum of their measure is 180° .

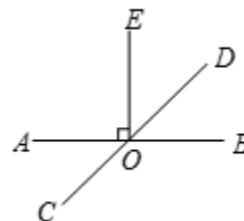
Linear Pairs: A linear pair is a pair of adjacent angles formed when two lines intersect. The sum of their measures is 180.



Math Exploration 1

As shown in the figure below, line AB meets line CD at O . $OE \perp AB$ and $\angle BOD = 45^\circ$.

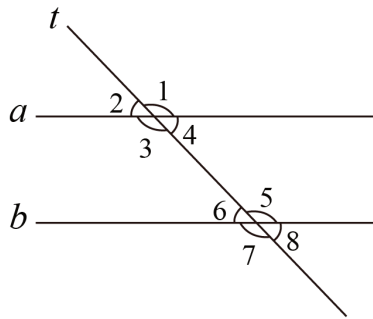
- (1) $\angle COE = \underline{\hspace{2cm}}^\circ$, it is an $\underline{\hspace{2cm}}$ (obtuse/acute) angle.
- (2) $\angle COA = \underline{\hspace{2cm}}^\circ$, it is an $\underline{\hspace{2cm}}$ (obtuse/acute) angle.
- (3) $\angle COA$ and $\angle COE$ are $\underline{\hspace{2cm}}$ (complementary/supplementary) angles.





Concept 2: Parallel Lines

If two lines in a plane do not meet, we say that they are **parallel**. As shown in the figure below, line a is parallel to line b and we can denote it as $a \parallel b$.



When a transversal t intersects two lines a and b :

two angles are **corresponding angles** if they occupy corresponding positions ($\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$);

two angles are **alternate interior angles** if they lie between the two lines on opposite sides of the transversal ($\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$);

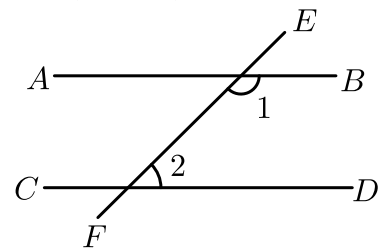
two angles are **same-side interior angles** which lie on the same side of the transversal and lie between two lines. ($\angle 4$ and $\angle 5$, $\angle 3$ and $\angle 6$).

If a transversal t intersects two **parallel lines** a and b , then **corresponding angles**, **alternate interior angles** are congruent and the sum of **same-side interior angles** is 180° .

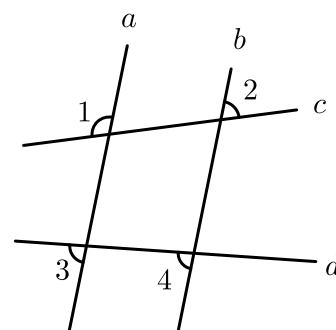
For example, $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$, $\angle 4 = \angle 8$; $\angle 3 = \angle 5$, $\angle 4 = \angle 6$; $\angle 4 + \angle 5 = 180^\circ$, $\angle 3 + \angle 6 = 180^\circ$.

Math Exploration 2

- 1 As shown in the figure below, $AB \parallel CD$ and $\angle 1 = (4x - 25)^\circ$, $\angle 2 = (85 - x)^\circ$. Find $\angle 1$.

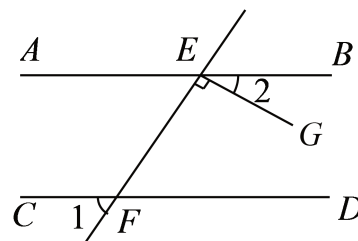


- 2 As shown in the figure below, $a \parallel b$, which of the following is true? ()



- A. $\angle 1 = \angle 2$ B. $\angle 3 + \angle 4 = 180^\circ$ C. $\angle 2 + \angle 4 = 180^\circ$ D. $\angle 1 + \angle 2 = 180^\circ$

- 3 As shown in the figure below, $AB \parallel CD$ and line EF meets AB and CD at E , F , respectively. $EG \perp EF$ at E , if $\angle 1 = 60^\circ$, then the measure of $\angle 2$ is () .



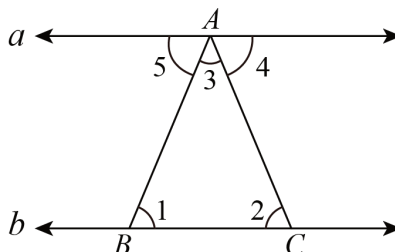
- A. 15° B. 30° C. 45° D. 60°



Concept 3: Angles of Triangles

1. Interior angles of the triangle: Angles which are formed by two sides of the triangle. A triangle has three interior angles on its three vertices.

The interior angle sum of a triangle is 180° .



Proof:

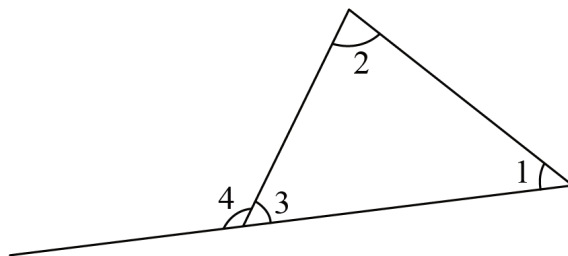
Draw line a such that $a \parallel b$ and line a intersects $\triangle ABC$ at point A . Side BC is on line b .

$$\angle 3 + \angle 4 + \angle 5 = 180^\circ$$

$\angle 1 = \angle 5$, $\angle 2 = \angle 4$ as they are alternate interior angles.

Thus, $\angle 1 + \angle 2 + \angle 3 = 180^\circ$, meaning that the interior angle sum of $\triangle ABC$ is 180° .

2.



Exterior angles of the triangle: Angles which are formed by one side of the triangle and the extension of another side.

Example: $\angle 4$

Remote interior angles: Interior angles which are not adjacent to the exterior angle.

Example: $\angle 1$ and $\angle 2$ are remote angles to $\angle 4$.

Angle relationships between these angles:

$$\angle 1 + \angle 2 + \angle 3 = 180^\circ$$

$$\angle 3 + \angle 4 = 180^\circ$$

We can subtract these two equations.

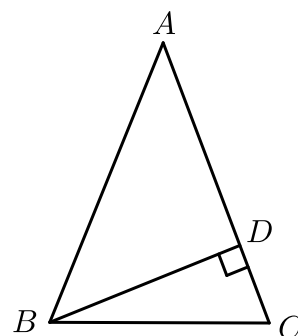
$$\angle 4 = \angle 1 + \angle 2$$

So we can summarize that the measure of an exterior angle is equal to the sum of its remote angles, and this is called The Exterior Angle Theorem.

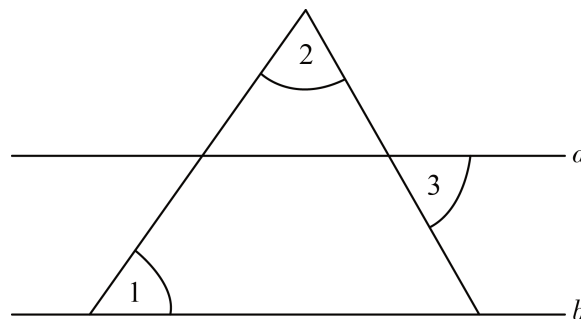
Math Exploration 3

- 1 In $\triangle ABC$, $\angle A = 35^\circ$, $\angle B = 90^\circ$ then the measure of $\angle C$ is () .
A. 55° B. 65° C. 75° D. 85°

- 2 As shown in the figure below, in $\triangle ABC$, $\angle C = \angle ABC = 2\angle A$. BD is perpendicular to AC , then $\angle DBC = \underline{\hspace{2cm}}$.



- 3 As shown in the figure below, $a \parallel b$, $\angle 1 = 55^\circ$, $\angle 2 = 65^\circ$, then the measure of $\angle 3$ is () .



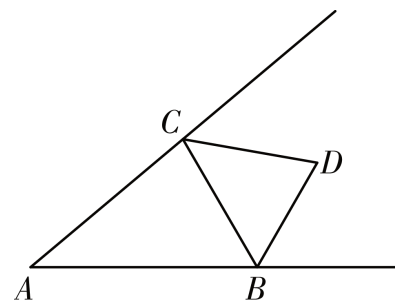
A. 50°

B. 55°

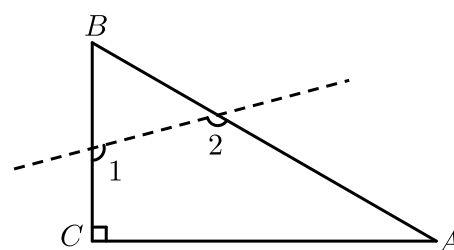
C. 60°

D. 65°

- 4 As shown in the figure below, given that the exterior angle bisectors of $\angle ABC$ and $\angle ACB$ meet at D , $\angle A = 40^\circ$. Find $\angle D$.



- 5 In right triangle $\triangle ABC$, $\angle B = 60^\circ$, then $\angle 1 + \angle 2$ is () .



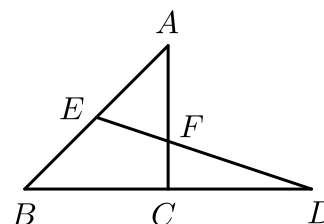
A. 150°

B. 180°

C. 240°

D. 300°

- 6 As shown in the figure below, $AC \perp BD$ at C , given that $\angle A = 40^\circ$ and $\angle AEF = 70^\circ$, then the measure of $\angle D$ is () .



A. 20°

B. 30°

C. 40°

D. 50°



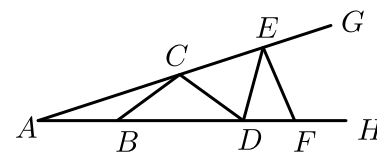
Concept 4: Find Angles in Complicated Figures

Steps to find the angle in complicated figures:

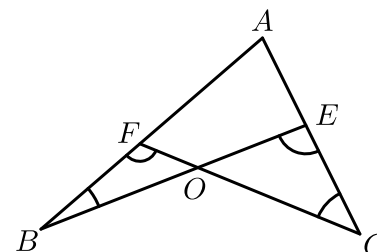
1. Observe the figure and suppose an angle be x
2. Use the properties of parallel lines and triangles to find the relationship between different angles
3. Make an equation based on the relationship and find x

Math Exploration 4

- 1 As shown in the figure below, C, E and B, D, F are on the two sides of $\angle GAH$, respectively. $AB = BC = CD = DE = EF$, if $\angle A = 18^\circ$, find $\angle GEF$.



- 2 As shown in the figure below, point E is on AC and point F is on AB . BE and CF intersect at O and $\angle C = 2\angle B$, $\angle BFC - \angle BEC = 20^\circ$. Find $\angle C$.



Lesson 2 Find the Length



Concept 1: The Pythagorean Theorem

1. The Pythagorean Theorem:

In a right triangle, the two sides that form the right angle are the legs, the other side is called hypotenuse.

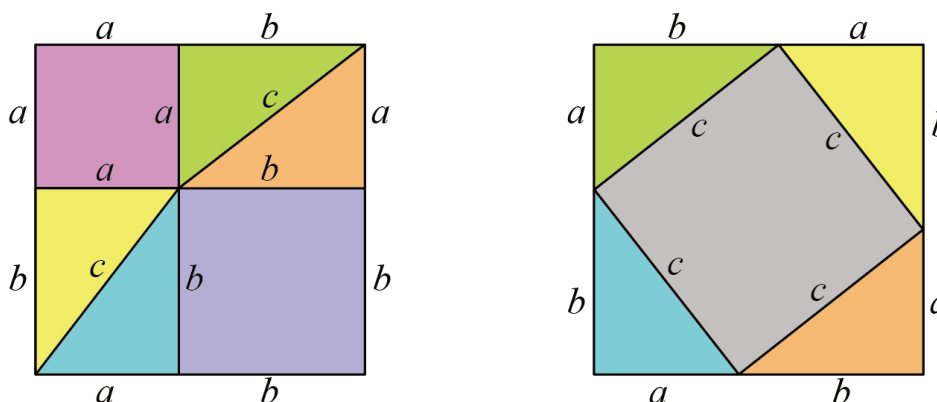
The Pythagorean Theorem is that, in a right triangle, two legs, a, b , and hypotenuse c satisfy that $a^2 + b^2 = c^2$.

We can prove it in this way:

(1) Firstly, draw a right triangle whose longer leg is b and shorter leg is a . c is the hypotenuse.

(2) Use four right triangles to form two squares as shown below.

(3) Since the area of square 1 is the same as that of square 2, we can get $a^2 + b^2 + 2ab = 2ab + c^2$, $a^2 + b^2 = c^2$.



2. The converse of the Pythagorean Theorem

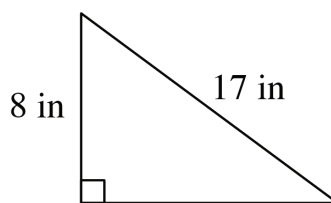
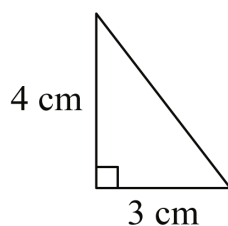
The converse of the Pythagorean Theorem mentions that if the side lengths of a triangle satisfy $a^2 + b^2 = c^2$, the triangle is a right triangle.

3. As the picture shown below, for example, if we want to find the distance between $A(x_1, y_1)$ and $B(x_2, y_2)$. we can find the point $C(x_2, y_1)$.

Then we can get $AC = |x_2 - x_1|$, $BC = |y_2 - y_1|$. According to the Pythagorean Theorem, we can get $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Math Exploration 1

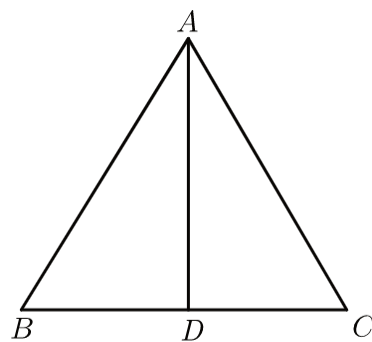
- 1 Find the length of the missing side:



- 2 In $\triangle ABC$, $AB = 12\text{cm}$, $BC = 16\text{cm}$ and $AC = 20\text{cm}$, then the area of $\triangle ABC$ is () .
A. 96cm^2 B. 120cm^2 C. 160cm^2 D. 200cm^2

- 3 Given that the coordinates of point P are $(9, -4)$ and $Q(2, 4)$, then the distance from P to point Q is _____ .

- 4 As shown in the figure below, in $\triangle ABC$, $AB = 10$, $BC = 12$, D is the midpoint of BC and $AD = 8$. The length of AC is ().



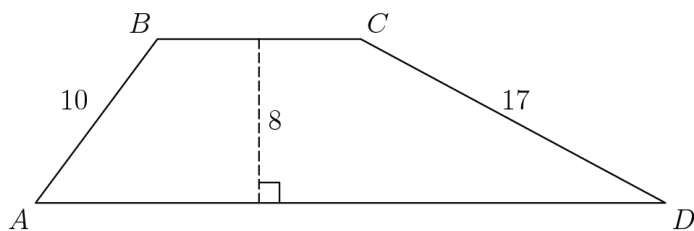
A. 10

B. 12

C. $2\sqrt{34}$

D. $4\sqrt{13}$

- 5 The area of trapezoid $ABCD$ is 164cm^2 . The altitude is 8cm, AB is 10cm, and CD is 17cm. What is BC , in centimeters? (2003 AMC 8 Problems, Question #21)



A. 9

B. 10

C. 12

D. 15

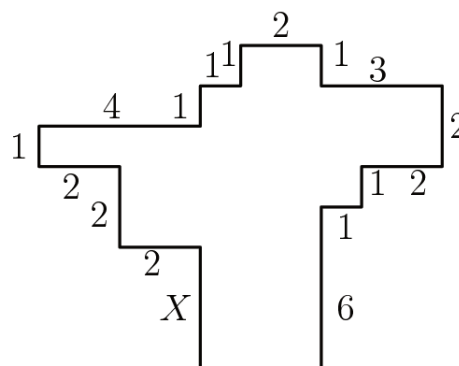
E. 20

Concept 2: Perimeter

The perimeter of a two-dimensional shape is the distance around the shape. We can use translation to find the perimeter of a complicated figure.

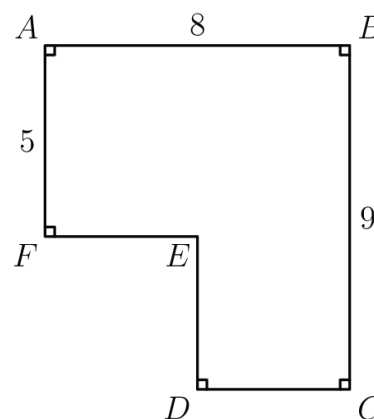
Math Exploration 2

- 1 In the diagram, all angles are right angles and the lengths of the sides are given in centimeters. Note the diagram is not drawn to scale. What is, X in centimeters? (2012 AMC 8 Problem, Question #5)



- A. 1 B. 2 C. 3 D. 4 E. 5

- 2 The area of polygon $ABCDEF$ is 52 with $AB = 8$, $BC = 9$ and $FA = 5$. What is $DE + EF$? (2005 AMC 8 Problem, Question #13)



- A. 7 B. 8 C. 9 D. 10 E. 11



Concept 3: Find Length in Triangles (Similar Figure)

1. **Similar figures** have the same shape but may have different sizes.

If two figures are **similar** ($\triangle ABC \sim \triangle A'B'C'$), we have:

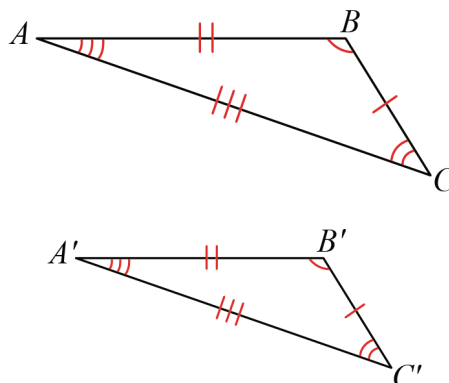
(1) their corresponding angles have the same measure ($\angle A = \angle A'$, $\angle B = \angle B'$, $\angle C = \angle C'$)

(2) the lengths of their corresponding sides are proportional

$$\left(\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'} = \frac{\text{Perimeter } ABC}{\text{Perimeter } A'B'C'} \right).$$

(3) the ratio of the area of two similar triangles is equal to the square of the ratio of any pair of the corresponding sides of the similar triangles.

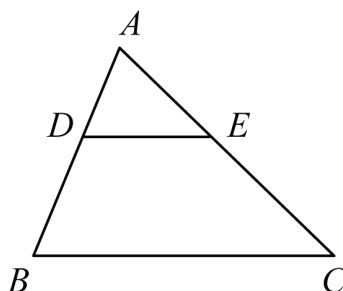
$$\left(\frac{\text{Area } \triangle ABC}{\text{Area } \triangle A'B'C'} = \frac{AB^2}{A'B'^2} = \frac{AC^2}{A'C'^2} = \frac{BC^2}{B'C'^2} \right)$$



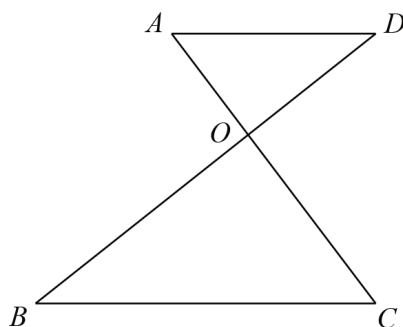
2. **Angle-Angle (AA) Similarity Postulate:** If two angles of one triangle are equal to two angles of another triangle, then the triangles are similar.

3. "A" model: If $DE \parallel BC$, we have $\triangle ADE \sim \triangle ABC$, and then

$$\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}.$$

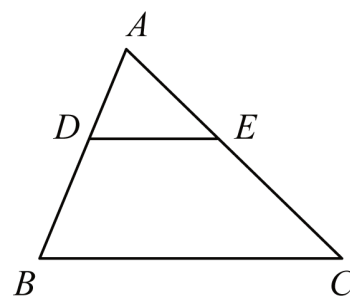


“8” model: If $AD \parallel BC$, we have $\triangle OAD \sim \triangle OCB$, and then $\frac{AD}{CB} = \frac{AO}{CO} = \frac{BO}{DO}$.

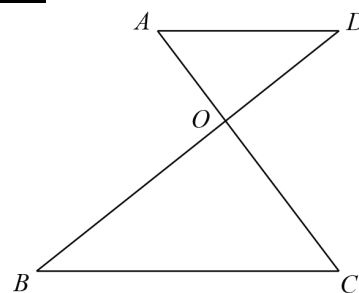


Math Exploration 3

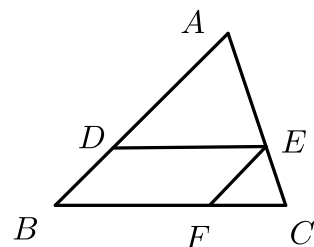
- 1 Given that $\overline{DE} \parallel \overline{BC}$, $AE : AC = 3 : 7$, $AD = 15$, $AC = 60$, $AE = \underline{\hspace{2cm}}$, $AB = \underline{\hspace{2cm}}$.



- 2 Given that $\overline{AD} \parallel \overline{BC}$, $OA = 12$, $OC = 18$, and $AD = 24$, $BC = \underline{\hspace{2cm}}$.

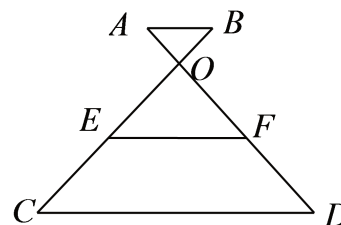


- 3 As shown in the figure below, in $\triangle ABC$, point D , E and F are on side AB , AC and BC , respectively and $DE \parallel BC$, $EF \parallel AB$. If $AD = 2BD$, then $\frac{CF}{CB}$ is () .



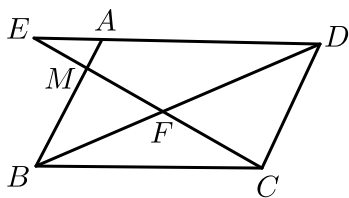
- A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{2}{3}$

- 4 As shown in the figure below, $AB \parallel CD \parallel EF$, $BO : OC = 1 : 4$, points E and F are midpoints of OC and OD , respectively. $EF : AB$ is () .



- A. 1 B. 2 C. 3 D. 4

- 5 As shown in the figure below, quadrilateral $ABCD$ is a parallelogram and point E is on line DA . Given that $AE = 2$, $AD = 8$ and $AM = 1$, find AB .



Blank lined paper with a green border and punch holes.

Lesson 3 Area and Perimeter



Concept 1: Areas of Basic Figures

Area of square: $A = l^2$

Area of rectangle: $A = lw$

Area of parallelogram: $A = bh$

Area of trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$

Area of triangle: $A = \frac{1}{2}bh$

Area of rhombus: $A = \frac{1}{2}d_1d_2$

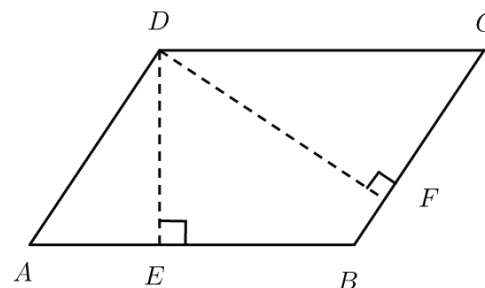
Area of circle: $A = \pi r^2$

Math Exploration 1

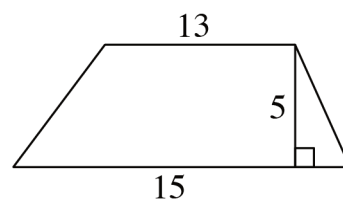
- 1 (1) In parallelogram $ABCD$, $\overline{AE}$ is perpendicular to $\overline{DC}$ at point E . $CD = 18$, $AE = 7$. The area of parallelogram $ABCD$ is _____.



- (2) In parallelogram $ABCD$, $\overline{DE}$ is perpendicular to $\overline{AB}$ at point E and $\overline{DF}$ is perpendicular to $\overline{BC}$ at point F . $AB = 21$, $DE = 16$, $DF = 24$, then $BC =$ _____.



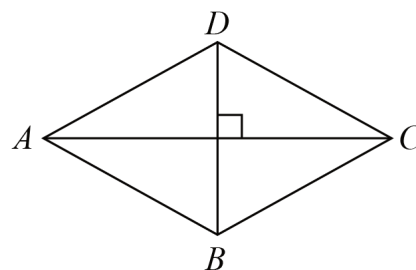
- 2 The area of the following trapezoid is _____ .



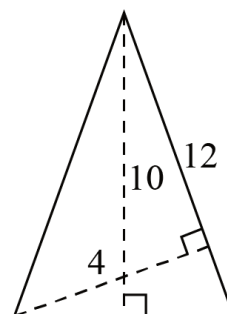
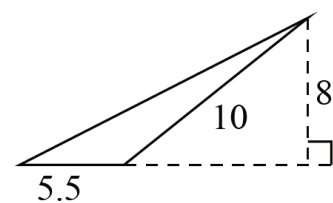
- 3 In rhombus $ABCD$, diagonal $AC = 48$ and diagonal $BD = 14$.

The area of rhombus $ABCD$ is _____ .

The side length of rhombus $ABCD$ is _____ .



- 4 Find the area of the triangles below.



- 5 1. If the height of a triangle is 12, and the area of the triangle is 36, the length of the corresponding base is _____ .

2. If the base of a triangle is 5, and the area of the triangle is 35, the length of the corresponding height is _____ .

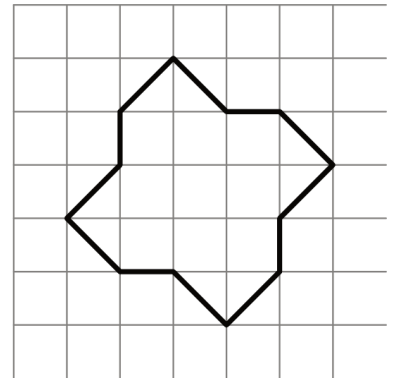


Concept 2: Areas of Shapes on Grids

To find the area on grid, cut the figure into several parts or patch them into a whole figure.

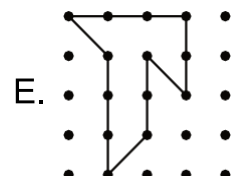
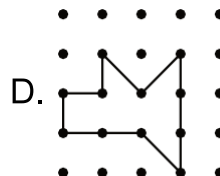
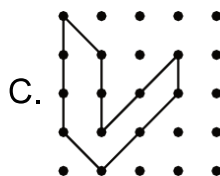
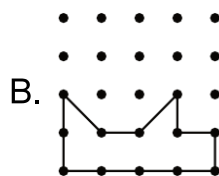
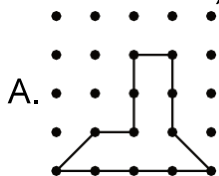
Math Exploration 2

- 1 The twelve-sided figure shown has been drawn on $1\text{cm} \times 1\text{cm}$ graph paper. What is the area of the figure in cm^2 ? (). (2018 AMC 8 Problem, Question #4)

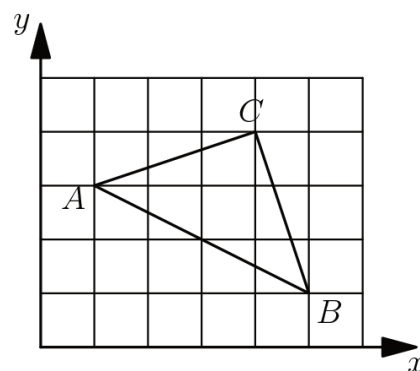


- A. 12 B. 12.5 C. 13 D. 13.5 E. 14

- 2 Which of the following polygons has the largest area? (). (2002 AMC 8 Problem, Question #15)



- 3 A triangle with vertices as $A = (1, 3)$, $B = (5, 1)$, and $C = (4, 4)$ is plotted on a 6×5 grid. What fraction of the grid is covered by the triangle? () . (2015 AMC 8 Problem, Question #19)



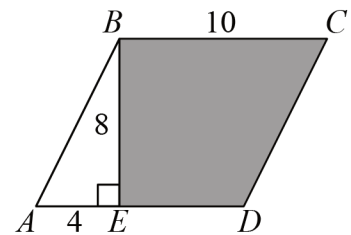
- A. $\frac{1}{6}$ B. $\frac{1}{5}$ C. $\frac{1}{4}$ D. $\frac{1}{3}$ E. $\frac{1}{2}$



Concept 3: Areas of Complicated Figures

Math Exploration 3

- 1 The area of the shaded region $BEDC$ in parallelogram $ABCD$ is (). (Adapted from 1989 AJHSME Problem, Question #15)



A. 24

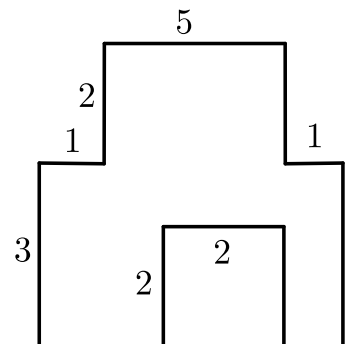
B. 48

C. 60

D. 64

E. 80

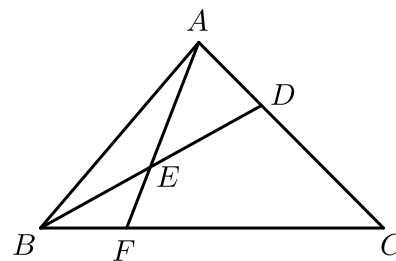
- 2 Find the area of the figure below.



- 3 A 1×2 rectangle is inscribed in a semicircle with longer side on the diameter. What is the area of the semicircle? () . (2013 AMC 8 Problem, Question #20)

A. $\frac{\pi}{2}$ B. $\frac{2\pi}{2}$ C. π D. $\frac{4\pi}{3}$ E. $\frac{5\pi}{3}$

- 4 In triangle ABC , point D divides side $\overline{AC}$ so that $AD : DC = 1 : 2$. Let E be the midpoint of $\overline{BD}$ and let F be the point of intersection of line BC and line AE . Given that the area of $\triangle ABC$ is 360, what is the area of $\triangle EBF$? (2019 AMC 8 Problem, Question #24)



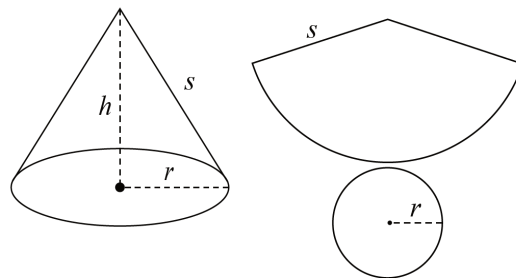
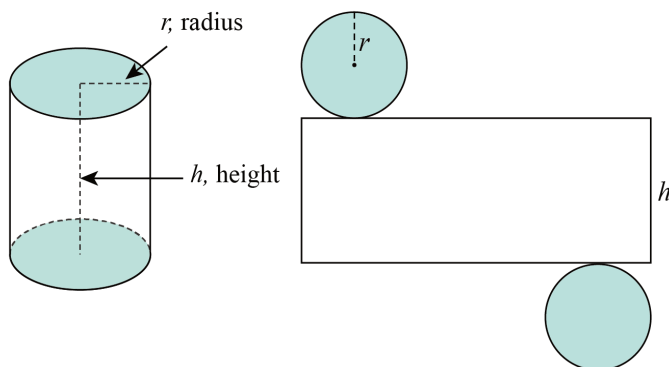
A. 24 B. 30 C. 32 D. 36 E. 40

Blank lined paper with a green border and punch holes.

Lesson 4 Volume



Concept 1: Volume of Cylinders and Cones



A cylinder is a solid with two congruent circular bases that lie in parallel lines.

$$\text{Volume } V = Bh = \pi r^2 h$$

A cone is a solid with a circular base.

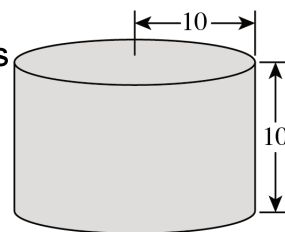
The slant height s of a cone is the distance between any point on the edge of the base and the vertex.

$$\text{Slant height } s = \sqrt{r^2 + h^2}$$

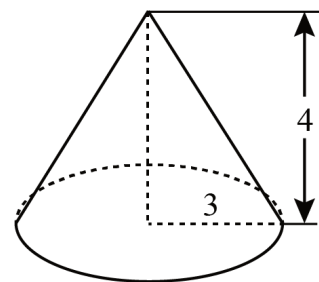
$$\text{Volume } V = \frac{1}{3} \pi r^2 h$$

Math Exploration 1

- 1 Answer the questions (leave π in the answer):
The height of a cylinder is 10, and the radius of the circular base is also 10.
The volume of the cylinder is _____.



- 2 Find the volume of the following cone. (keep π in the answer)



- 3 (1) If the volume of a cylinder is 375π and the height is 15π , the radius is _____ .

(2) If the surface area of a cylinder is 128π , and the radius is 4, the height is _____ .

- 4 (1) The cylinder and the cone have the same base and the same volume. If the height of the cylinder is 3 cm, then the height of the cone is _____ cm.

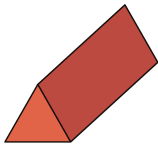
(2) The cylinder and the cone have the same volume and the same height. If the radius of the base of cylinder is 3 cm, then the radius of the base of cone is _____ cm.

- 5 Alex and Felicia each have cats as pets. Alex buys cat food in cylindrical cans that are 8 cm in diameter and 12 cm high. Felicia buys cat food in cylindrical cans that are 12 cm in diameter and 8 cm high. What is the ratio of the volume one of Alex's cans to the volume one of Felicia's cans?
- 6 A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased? (2004 AMC 12A Problem, Question #9)
- A. 10 B. 25 C. 36 D. 50 E. 60

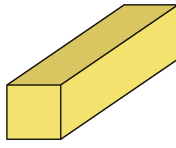


Concept 2: Volume of Prisms and Pyramids

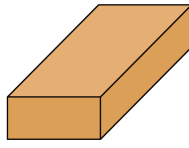
1. Definition of Prism: A prism is a three-dimensional figure with two identical and parallel bases that are polygons and the other faces are rectangles. A prism is identified by the shape and its base.



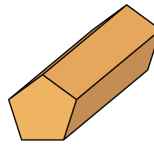
triangular prism



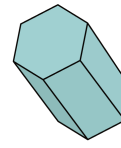
square prism



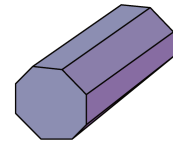
rectangular prism



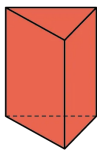
pentagonal prism



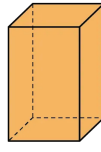
hexagonal prism



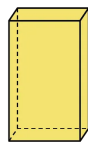
octagonal prism



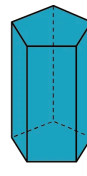
triangular prism



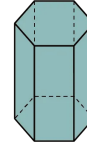
square prism



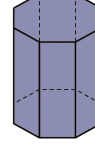
rectangular prism



pentagonal prism



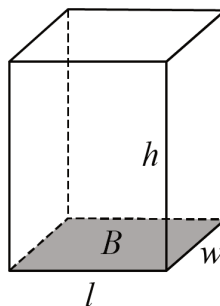
hexagonal prism



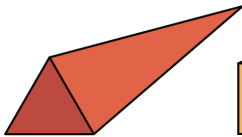
octagonal prism

2. Volume of prisms:

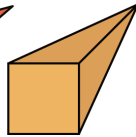
$$V = Bh$$



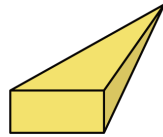
3. A pyramid is a three-dimensional figure whose base is a polygon and whose other faces are triangles that meet at a point. A pyramid is identified by the shape of its base.



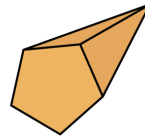
triangular pyramid



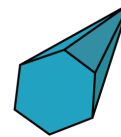
square pyramid



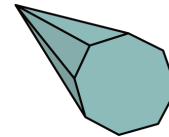
rectangular pyramid



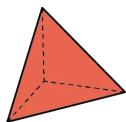
pentagonal pyramid



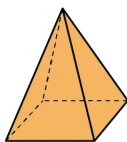
hexagonal pyramid



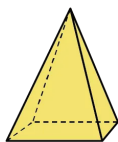
octagonal pyramid



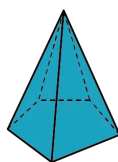
triangular pyramid



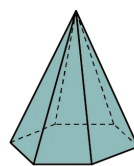
square pyramid



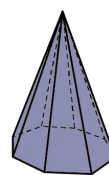
rectangular pyramid



pentagonal pyramid



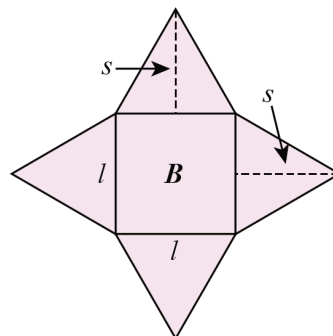
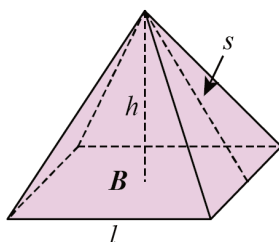
hexagonal pyramid



octagonal pyramid

4. Volume of pyramids:

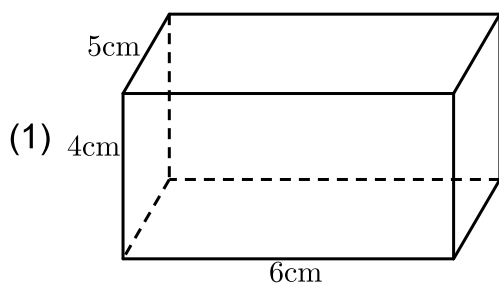
Surface Area of Pyramid



The volume is calculated by $\frac{1}{3} \times B \times h$.

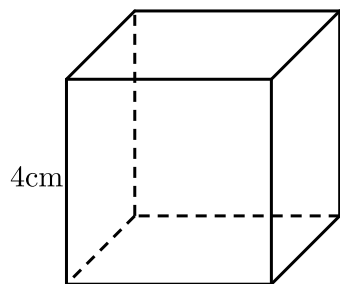
Math Exploration 2

- 1 Find the volumes of the following solid figures:



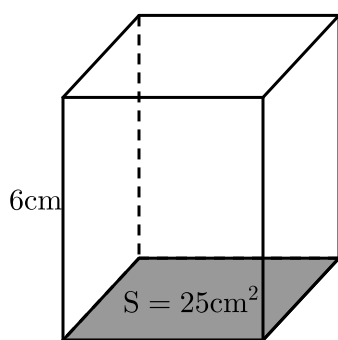
$V =$ _____ cm^3 .

(2)



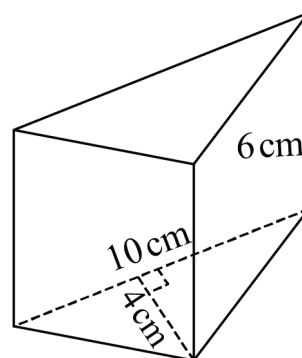
$V =$ _____ cm^3 .

(3)



$V =$ _____ cm^3 .

2 Find the volume of the following triangular prism:



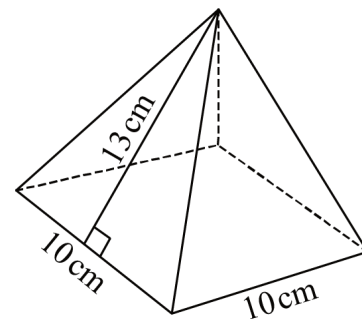
- 3 Find the surface area of the square pyramid.

The area of the base is _____ cm^2 .

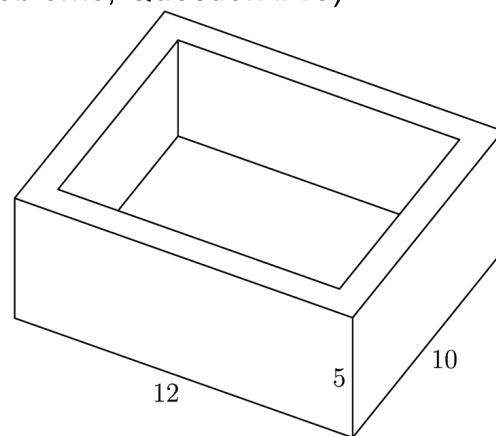
The area of the four faces is _____ cm^2 .

The surface area is _____ cm^2 .

The volume is _____ cm^3 .

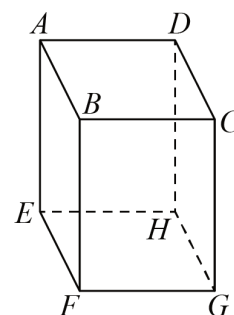


- 4 Isabella uses one-foot cubical blocks to build a rectangular fort that is 12 feet long, 10 feet wide, and 5 feet high. The floor and the four walls are all one foot thick. How many blocks does the fort contain? (). (2018 AMC 8 Problems, Question #18)



- A. 204 B. 280 C. 320 D. 340 E. 600

- 5 In following rectangular prism, $AB = 4$, $BC = 3$, $BF = 6$. Find the volume of triangular prism $ABC - EFG$ and triangular pyramid $A - EFG$ and the length of AG .

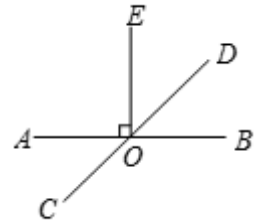


Learning Modules

Lesson 1

1 As shown in the figure below, line AB meets line CD at O . $OE \perp AB$ and $\angle BOD = 45^\circ$.

- (1) $\angle COE = \underline{\hspace{2cm}}^\circ$, it is an $\underline{\hspace{2cm}}$ (obtuse/acute) angle.
- (2) $\angle COA = \underline{\hspace{2cm}}^\circ$, it is an $\underline{\hspace{2cm}}$ (obtuse/acute) angle.
- (3) $\angle COA$ and $\angle COE$ are $\underline{\hspace{2cm}}$ (complementary/supplementary) angles.



Answer 1:135

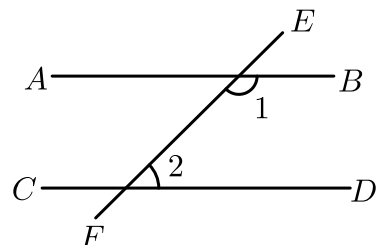
2:obtuse

3:45

4:acute

5:supplementary

2 As shown in the figure below, $AB \parallel CD$ and $\angle 1 = (4x - 25)^\circ$, $\angle 2 = (85 - x)^\circ$. Find $\angle 1$.



Answer 135°

Solution

$$\therefore AB \parallel CD,$$

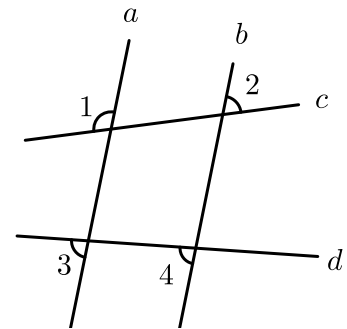
$$\therefore \angle 1 + \angle 2 = 180^\circ,$$

$$\text{we can get } (4x - 25) + (85 - x) = 180,$$

$$\text{then } x = 40.$$

$$\therefore \angle 1 = (4x - 25)^\circ = 135^\circ.$$

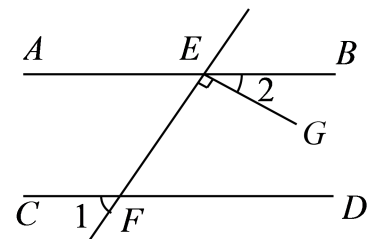
- 3 As shown in the figure below, $a \parallel b$, which of the following is true? ()



- A. $\angle 1 = \angle 2$ B. $\angle 3 + \angle 4 = 180^\circ$ C. $\angle 2 + \angle 4 = 180^\circ$ D. $\angle 1 + \angle 2 = 180^\circ$

Answer D

- 4 As shown in the figure below, $AB \parallel CD$ and line EF meets AB and CD at E , F , respectively. $EG \perp EF$ at E , if $\angle 1 = 60^\circ$, then the measure of $\angle 2$ is () .



- A. 15° B. 30° C. 45° D. 60°

Answer B

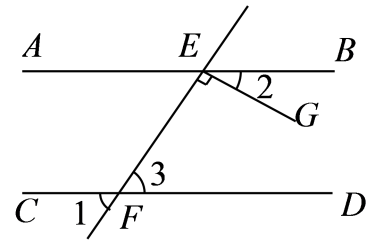
Solution $\angle 3 = \angle 1 = 60^\circ$,

$$\therefore AB \parallel CD, EG \perp EF,$$

$$\therefore \angle 3 + 90^\circ + \angle 2 = 180^\circ,$$

$$60^\circ + 90^\circ + \angle 2 = 180^\circ,$$

$$\angle 2 = 30^\circ.$$



5 In $\triangle ABC$, $\angle A = 35^\circ$, $\angle B = 90^\circ$ then the measure of $\angle C$ is () .

A. 55°

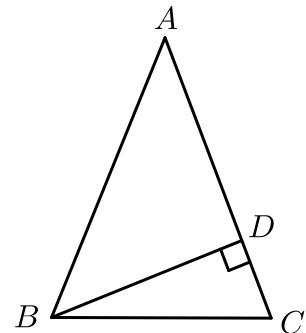
B. 65°

C. 75°

D. 85°

Answer A

6 As shown in the figure below, in $\triangle ABC$, $\angle C = \angle ABC = 2\angle A$. BD is perpendicular to AC , then $\angle DBC = \underline{\hspace{2cm}}$.



Answer 18°

Solution Suppose $\angle A = x$, then $\angle C = \angle ABC = 2x$.

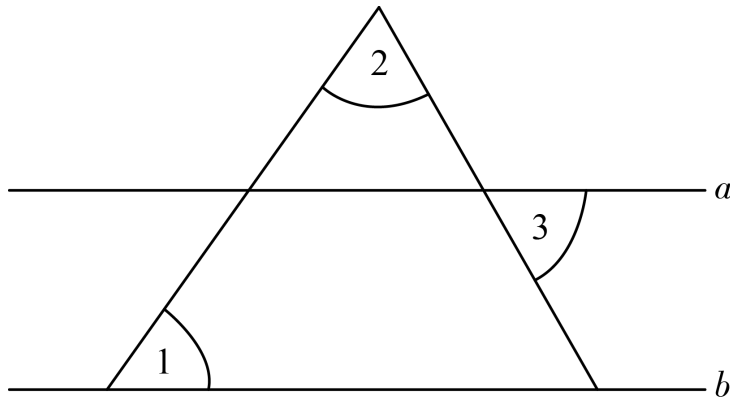
$$\angle C + \angle ABC + \angle A = 180^\circ,$$

$$\text{so } 2x + 2x + x = 180^\circ,$$

$$\text{therefore } x = 36^\circ, \angle C = 2x = 72^\circ.$$

$$\text{In right triangle } BDC, \angle DBC = 90^\circ - \angle C = 90^\circ - 72^\circ = 18^\circ.$$

As shown in the figure below, $a \parallel b$, $\angle 1 = 55^\circ$, $\angle 2 = 65^\circ$, then the measure of $\angle 3$ is () .



A. 50°

B. 55°

C. 60°

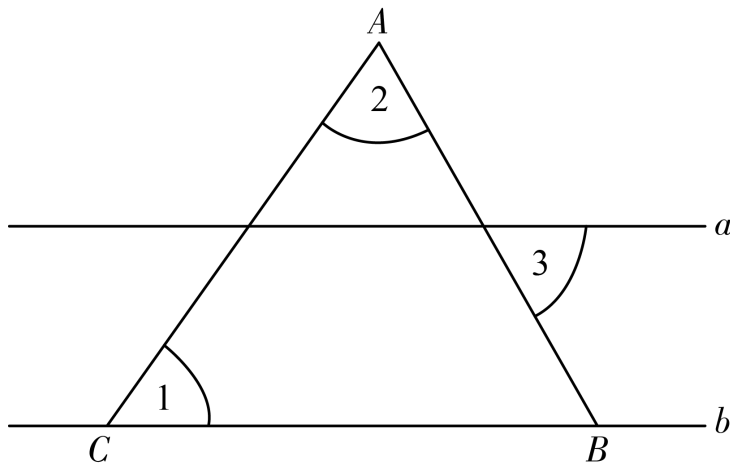
D. 65°

Answer C

Solution As shown in the figure below,

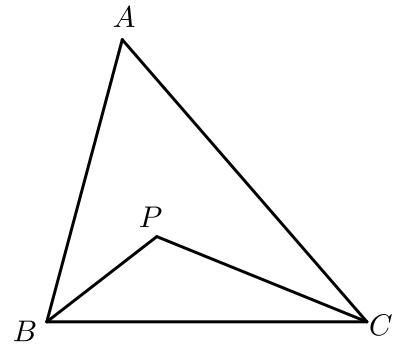
$$\because \angle 1 = 55^\circ, \angle 2 = 65^\circ, \therefore \angle ABC = 60^\circ.$$

$$\because a \parallel b, \therefore \angle 3 = \angle ABC = 60^\circ.$$



8

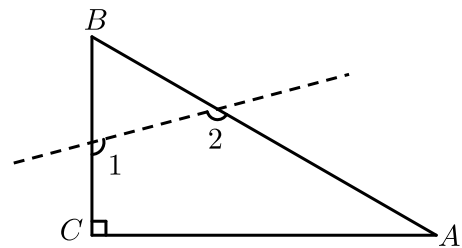
As shown in the figure below, in $\triangle ABC$, $\angle BAC = 50^\circ$, BP bisects $\angle ABC$ and CP bisects $\angle ACB$. Then $\angle BPC$ is _____ $^\circ$.



Answer 115

Solution $\angle BPC = \angle A + \angle ABP + \angle ACP$
 $= (180^\circ - \angle ABC - \angle ACB) + \frac{1}{2}(\angle ABC + \angle ACB)$
 $= 115^\circ$

9 In right triangle $\triangle ABC$, $\angle B = 60^\circ$, then $\angle 1 + \angle 2$ is ().



A. 150°

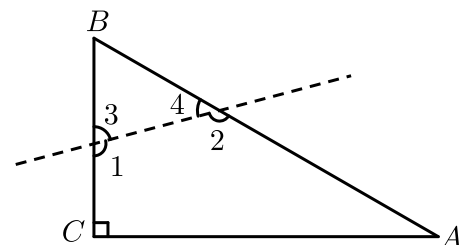
B. 180°

C. 240°

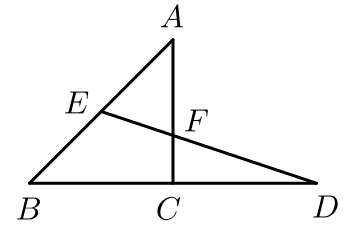
D. 300°

Answer C

Solution $\angle 1 = \angle B + \angle 4$, $\angle 2 = \angle 3 + \angle B$,
 $\therefore \angle 1 + \angle 2 = \angle 3 + \angle 4 + 2\angle B = 180^\circ + \angle B = 240^\circ$.



10 As shown in the figure below, $AC \perp BD$ at C , given that $\angle A = 40^\circ$ and $\angle AEF = 70^\circ$, then the measure of $\angle D$ is ().



A. 20°

B. 30°

C. 40°

D. 50°

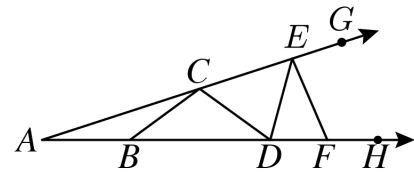
Answer A

Solution $\angle AFE = 180^\circ - \angle A - \angle AEF = 180^\circ - 40^\circ - 70^\circ = 70^\circ$,

$$\angle CFD = \angle AFE = 70^\circ,$$

$$\angle D = 180^\circ - \angle CFD - \angle ACD = 180^\circ - 70^\circ - 90^\circ = 20^\circ.$$

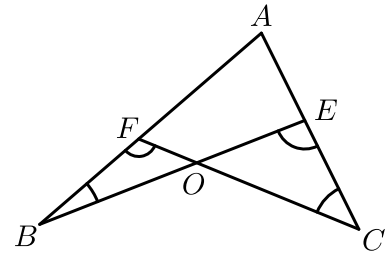
- 11 As shown in the figure below, C, E and B, D, F are on the two sides of $\angle GAH$, respectively. $AB = BC = CD = DE = EF$, if $m\angle A = 18^\circ$, find $m\angle GEF$.



Answer $m\angle GEF = 90^\circ$

Solution $m\angle A = 18^\circ \rightarrow m\angle C = 18^\circ \rightarrow m\angle ABC = 180^\circ - 18^\circ - 18^\circ = 144^\circ \rightarrow m\angle CBD = 36^\circ$
 $\rightarrow m\angle BCD = 180^\circ - 36^\circ - 36^\circ = 108^\circ \rightarrow m\angle ECD = 180^\circ - 18^\circ - 108^\circ = 54^\circ \rightarrow m\angle EDC$
 $= 72^\circ \rightarrow m\angle EDF = m\angle EFD = 180^\circ - 72^\circ - 36^\circ = 72^\circ \rightarrow m\angle AEF = 180^\circ - 18^\circ - 72^\circ$
 $= 90^\circ \rightarrow m\angle GEF = 90^\circ$

- 12 As shown in the figure below, point E is on AC and point F is on AB . BE and CF intersect at O and $\angle C = 2\angle B$, $\angle BFC - \angle BEC = 20^\circ$. Find $\angle C$.



Answer 40°

Solution $\because \angle BFC = \angle A + \angle C,$

$$\angle BEC = \angle A + \angle B,$$

$$\therefore \angle BFC - \angle BEC = \angle C - \angle B = 20^\circ,$$

$$\therefore \angle C = 2\angle B,$$

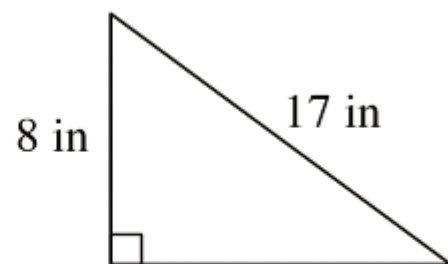
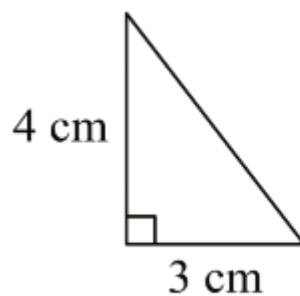
$$\therefore 2\angle B - \angle B = 20^\circ,$$

$$\therefore \angle B = 20^\circ,$$

$$\therefore \angle C = 2\angle B = 40^\circ.$$

Lesson 2

13 Find the length of the missing side:



Answer 5 cm; 8 inches.

14 In $\triangle ABC$, $AB = 12\text{cm}$, $BC = 16\text{cm}$ and $AC = 20\text{cm}$, then the area of $\triangle ABC$ is () .

A. 96cm^2

B. 120cm^2

C. 160cm^2

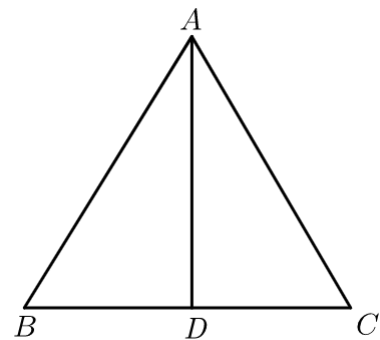
D. 200cm^2

Answer A

- 15 Given that the coordinates of point P are $(9, -4)$ and $Q(2, 4)$, then the distance from P to point Q is _____.

Answer $\sqrt{113}$

- 16 As shown in the figure below, in $\triangle ABC$, $AB = 10$, $BC = 12$, D is the midpoint of BC and $AD = 8$. The length of AC is ().



A. 10

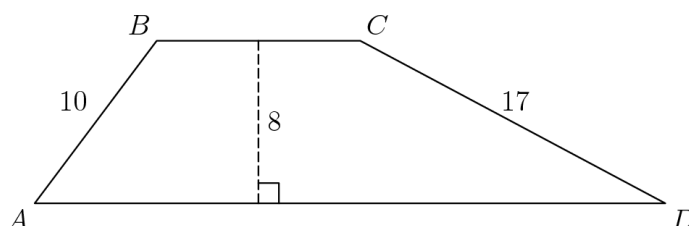
B. 12

C. $2\sqrt{34}$

D. $4\sqrt{13}$

Answer A

- 17 The area of trapezoid $ABCD$ is 164cm^2 . The altitude is 8cm , AB is 10cm , and CD is 17cm . What is BC , in centimeters? (2003 AMC 8 Problems, Question #21)



A. 9

B. 10

C. 12

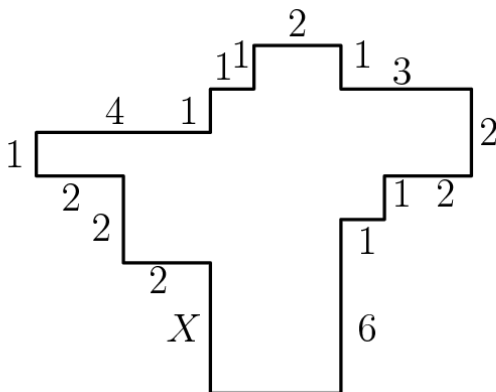
D. 15

E. 20

Answer B

Solution Using the formula for the area of a trapezoid, we have $164 = 8 \left(\frac{BC + AD}{2} \right)$. Thus $BC + AD = 41$. Drop perpendiculars from B to AD and from C to AD and let them hit AD at E and F respectively. Note that each of these perpendiculars has length 8. From the Pythagorean Theorem, $AE = 6$ and $DF = 15$ thus $AD = BC + 21$. Substituting back into our original equation we have $BC + BC + 21 = 41$ thus $BC =$ B) 10.

- 18 In the diagram, all angles are right angles and the lengths of the sides are given in centimeters. Note the diagram is not drawn to scale. What is, X in centimeters? (2012 AMC 8 Problem, Question #5)



A. 1

B. 2

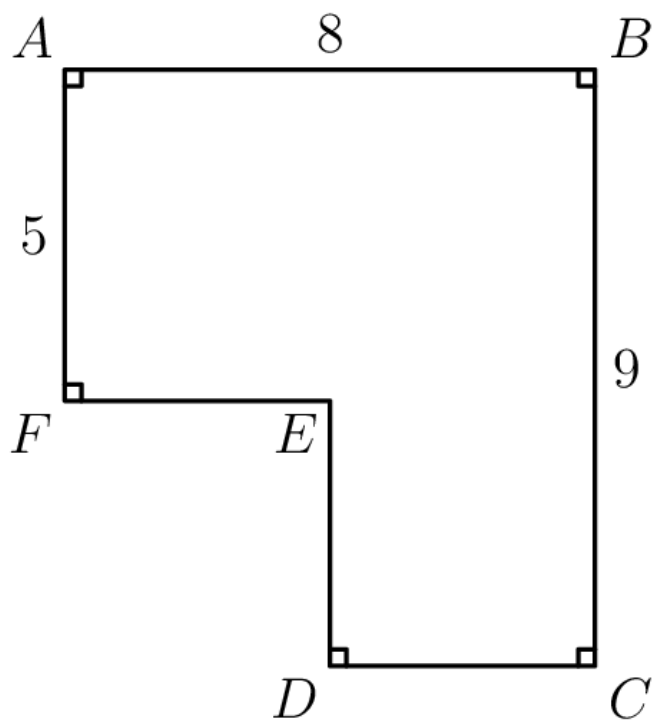
C. 3

D. 4

E. 5

Answer E

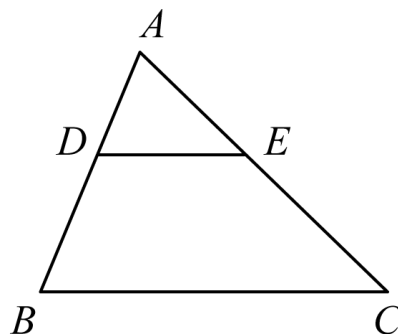
- 19 The area of polygon $ABCDEF$ is 52 with $AB = 8$, $BC = 9$ and $FA = 5$. What is $DE + EF$? (2005 AMC 8 Problem, Question #13)



- A. 7 B. 8 C. 9 D. 10 E. 11

Answer C

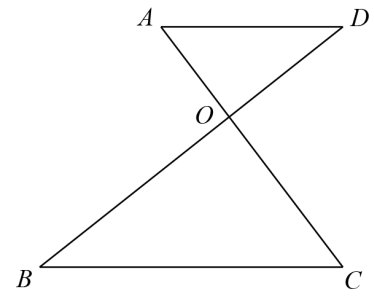
- 20 Given that $\overline{DE} \parallel \overline{BC}$, $AE : AC = 3 : 7$, $AD = 15$, $AC = 60$, $AE = \underline{\hspace{1cm}}$, $AB = \underline{\hspace{1cm}}$.



Answer 1:18

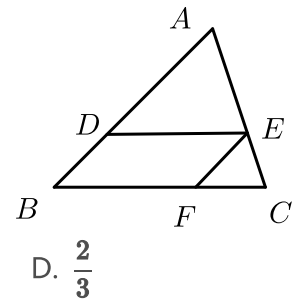
2:50

- 21 Given that $\overline{AD} \parallel \overline{BC}$, $OA = 12$, $OC = 18$, and $AD = 24$, $BC = \underline{\hspace{1cm}}$.



Answer 36

- 22 As shown in the figure below, in $\triangle ABC$, point D , E and F are on side AB , AC and BC , respectively and $DE \parallel BC$, $EF \parallel AB$. If $AD = 2BD$, then $\frac{CF}{CB}$ is ().



- A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{2}{3}$

Answer B

Solution $\because AD = 2BD$,

$$\therefore BD : AB = 1 : 3,$$

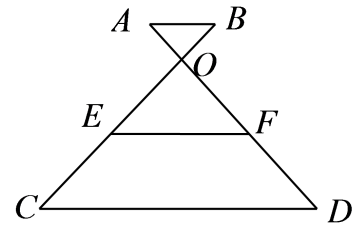
$$\because DE \parallel BC,$$

$$\therefore CE : AC = BD : AB = 1 : 3,$$

$$\because EF \parallel AB,$$

$$\therefore CF : CB = CE : AC = 1 : 3.$$

- 23 As shown in the figure below, $AB \parallel CD \parallel EF$, $BO : OC = 1 : 4$, points E and F are midpoints of OC and OD , respectively. $EF : AB$ is ().



A. 1 : 1

B. 2 : 1

C. 3 : 1

D. 4 : 1

Answer B

Solution $EF \parallel CD$

$$\therefore AB \parallel CD,$$

$$\therefore \triangle ABO \sim \triangle FEO,$$

$$\therefore EF : AB = EO : BO$$

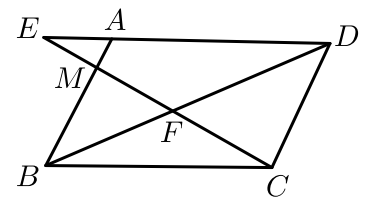
$$BO : OC = 1 : 4$$

$$\therefore OE = \frac{1}{2}OC$$

$$\therefore OE = 2OB$$

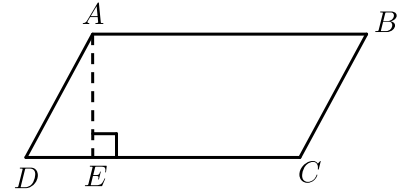
$$\therefore EF : AB = 2 : 1.$$

- 24 As shown in the figure below, quadrilateral $ABCD$ is a parallelogram and point E is on line DA . Given that $AE = 2$, $AD = 8$ and $AM = 1$, find AB .

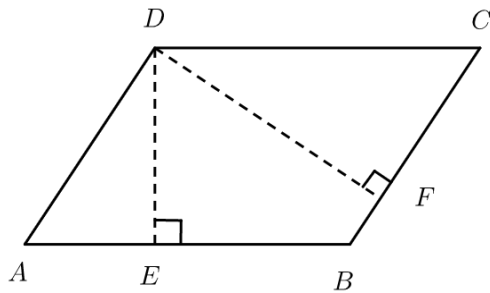


Answer 5

- 25 (1) In parallelogram $ABCD$, $\overline{AE}$ is perpendicular to $\overline{DC}$ at point E . $CD = 18$, $AE = 7$. The area of parallelogram $ABCD$ is _____ .

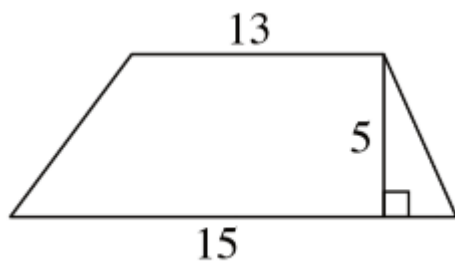


- (2) In parallelogram $ABCD$, $\overline{DE}$ is perpendicular to $\overline{AB}$ at point E and $\overline{DF}$ is perpendicular to $\overline{BC}$ at point F . $AB = 21$, $DE = 16$, $DF = 24$, then $BC =$ _____ .



Answer 1:126
2:14

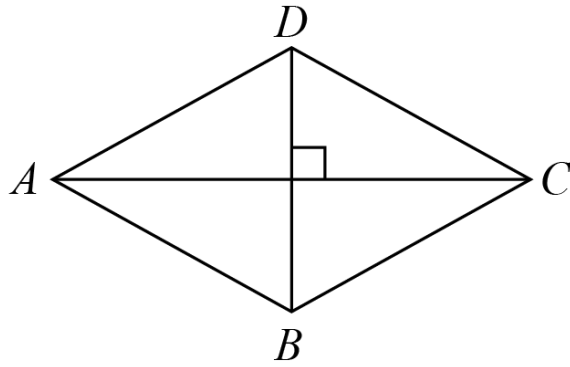
- 26 The area of the following trapezoid is _____ .



Answer 70

- 27 In rhombus $ABCD$, diagonal $AC = 48$ and diagonal $BD = 14$. The area of rhombus $ABCD$ is _____ .

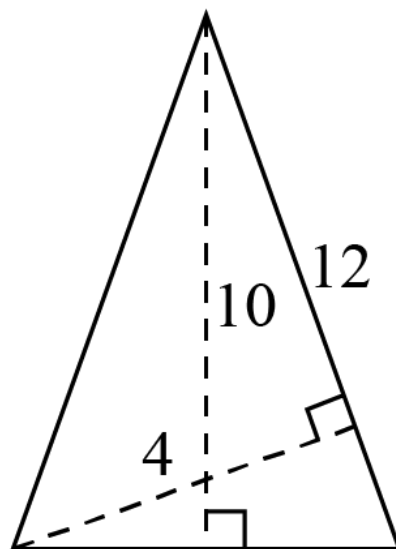
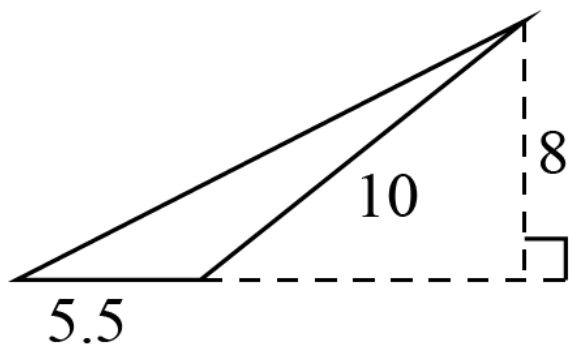
The side length of rhombus $ABCD$ is _____.



Answer 1:336

2:25

28 Find the area of the triangles below.



Answer 22

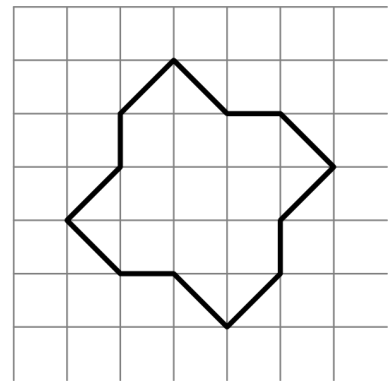
24

- 29
1. If the height of a triangle is **12**, and the area of the triangle is **36**, the length of the corresponding base is _____ .
 2. If the base of a triangle is **5**, and the area of the triangle is **35**, the length of the corresponding height is _____ .

Answer 1:6

2:14

- 30
- The twelve-sided figure shown has been drawn on **1cm × 1cm** graph paper. What is the area of the figure in **cm²**? () . (2018 AMC 8 Problem, Question #4)



A. 12

B. 12.5

C. 13

D. 13.5

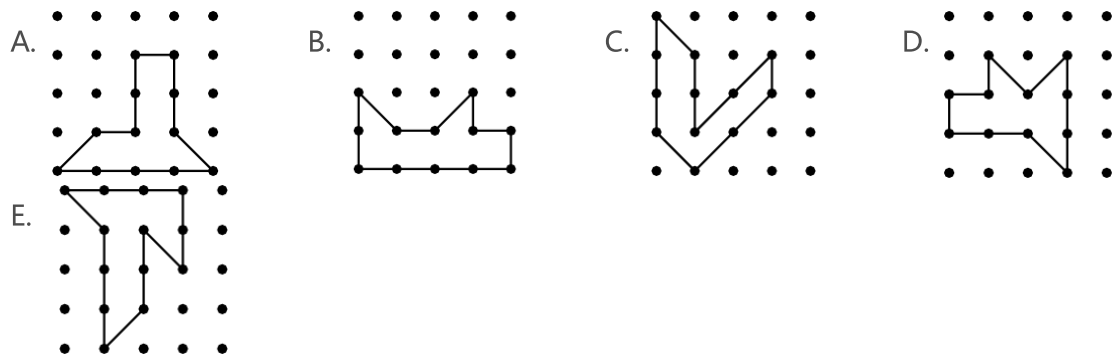
E. 14

Answer C

Solution We count $3 \cdot 3 = 9$ unit squares in the middle, and 4 small triangles each with an area of 1. Thus, the answer is $9 + 4 = \boxed{\text{(C)}13}$.

31

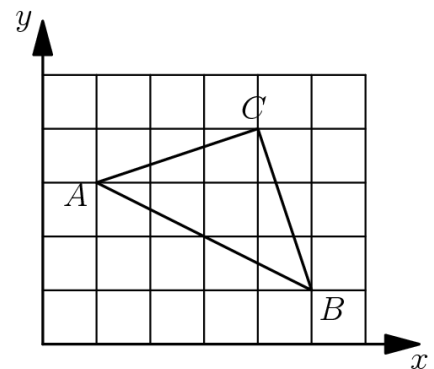
Which of the following polygons has the largest area? (). (2002 AMC 8 Problem, Question #15)



Answer E

Solution Each polygon can be divided into unit squares and right triangles with side length 1. Count the number of boxes enclosed by each polygon, with the unit square being 1, and the triangle being $\frac{1}{2}$. **A** has 5, **B** has 5, **C** has 5, **D** has 4.5, and **E** has 5.5. Therefore, the polygon with the largest area is **E**.

- 32 A triangle with vertices as $A = (1, 3)$, $B = (5, 1)$, and $C = (4, 4)$ is plotted on a 6×5 grid. What fraction of the grid is covered by the triangle? (). (2015 AMC 8 Problem, Question #19)



- A. $\frac{1}{6}$ B. $\frac{1}{5}$ C. $\frac{1}{4}$ D. $\frac{1}{3}$ E. $\frac{1}{2}$

Answer A

Solution

The area of $\triangle ABC$ is equal to half the product of its base and height. By the Pythagorean Theorem, we find its height is $\sqrt{1^2 + 2^2} = \sqrt{5}$, and its base is $\sqrt{2^2 + 4^2} = \sqrt{20}$. We multiply these and divide by 2 to find the area of the triangle is $\frac{\sqrt{5 \cdot 20}}{2} = \frac{\sqrt{100}}{2} = \frac{10}{2} = 5$. Since the grid has an area of 30, the fraction of the grid covered by the triangle $\frac{5}{30} = \boxed{(A) \frac{1}{6}}$.

Note angle $\angle ACB$ is right, thus the area is $\sqrt{1^2 + 3^2} \times \sqrt{1^2 + 3^2} \times \frac{1}{2} = 10 \times \frac{1}{2} = 5$ thus the fraction of the total is $\frac{5}{30} = \boxed{(A) \frac{1}{6}}$.

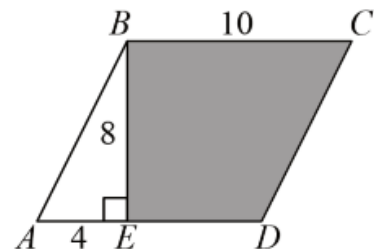
By the Shoelace theorem, the area of

$\triangle ABC = \left| \frac{1}{2}(15 + 4 + 4 - 1 - 20 - 12) \right| = \left| \frac{1}{2}(-10) \right| = 5$. This means the fraction of the total area is $\frac{5}{30} = \boxed{(A) \frac{1}{6}}$.

The smallest rectangle that follows the grid lines and completely encloses $\triangle ABC$ has an area of 12, where $\triangle ABC$ splits the rectangle into four triangles. The area of $\triangle ABC$ is therefore $12 - \left(\frac{4 \cdot 2}{2} + \frac{3 \cdot 1}{2} + \frac{3 \cdot 1}{2} \right) = 12 - \left(4 + \frac{3}{2} + \frac{3}{2} \right) = 12 - 7 = 5$. that means that $\triangle ABC$ takes up $\frac{5}{30} = \boxed{(A) \frac{1}{6}}$ of the grid.

Using Pick's Theorem, the area of the triangle is $4 + \frac{4}{2} - 1 = 5$. Therefore, the triangle takes up $\frac{5}{30} = \boxed{(A) \frac{1}{6}}$ of the grid.

- 33 The area of the shaded region $BEDC$ in parallelogram $ABCD$ is (). (Adapted from 1989 AJHSME Problem, Question #15)



- A. 24 B. 48 C. 60 D. 64 E. 80

Answer D

Solution Let $[ABC]$ denote the area of figure ABC .

Clearly, $[BEDC] = [ABCD] - [ABE]$. Using basic area formulas,

$$[ABCD] = (BC)(BE) = 80$$

$$[ABE] = \frac{(BE)(AE)}{2} = 4(AE)$$

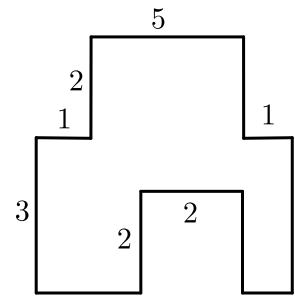
since $AE + ED = BC = 10$ and $ED = 6$, $AE = 4$ and the area of $\triangle ABE$ is $4(4) = 16$.

Finally, we have $[BEDC] = 80 - 16 = 64 \rightarrow \mathbf{D}$

Notice that $BEDC$ is a trapezoid. Therefore its area is

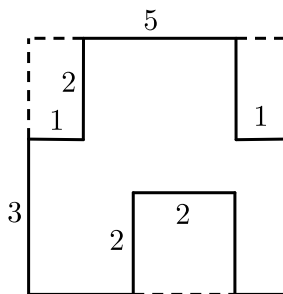
$$8 \left(\frac{6 + 10}{2} \right) = 8 \left(\frac{16}{2} \right) = 8(8) = 64 \Rightarrow \mathbf{D}.$$

34 Find the area of the figure below.



Answer 27

Solution



35 A 1×2 rectangle is inscribed in a semicircle with longer side on the diameter. What is the area of the semicircle? () . (2013 AMC 8 Problem, Question #20)

A. $\frac{\pi}{2}$

B. $\frac{2\pi}{2}$

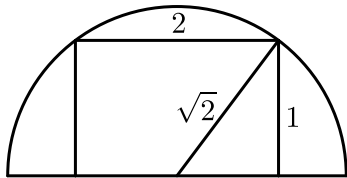
C. π

D. $\frac{4\pi}{3}$

E. $\frac{5\pi}{3}$

Answer C

Solution



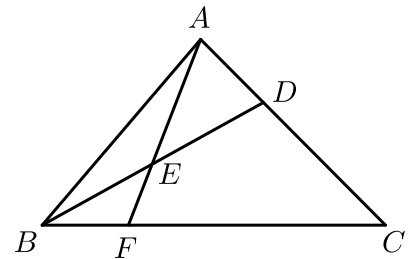
A semicircle has symmetry, so the center is exactly at the midpoint of the 2 side on the rectangle, making the radius, by the Pythagorean Theorem, $\sqrt{1^2 + 1^2} = \sqrt{2}$. The area is $\frac{2\pi}{2} = \pi$.

Double the figure to get a square with side length 2. The circle inscribed around the square has a diameter equal to the diagonal of this square. The diagonal of this square is $\sqrt{2^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$. The circle's radius, therefore, is $\sqrt{2}$.

The area of the circle is $\sqrt{2}^2 \cdot \pi = 2\pi$.

Finally, the area of the semicircle is π , so the answer is.

- 36 In triangle ABC , point D divides side $\overline{AC}$ so that $AD : DC = 1 : 2$. Let E be the midpoint of $\overline{BD}$ and let F be the point of intersection of line BC and line AE . Given that the area of $\triangle ABC$ is 360, what is the area of $\triangle EBF$? (2019 AMC 8 Problem, Question #24)



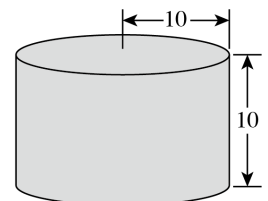
- A. 24 B. 30 C. 32 D. 36 E. 40

Answer B

Lesson 4

- 37 Answer the questions (leave π in the answer):

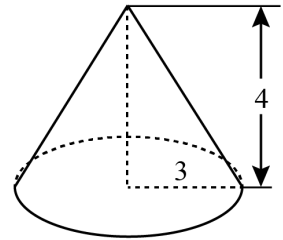
The height of a cylinder is 10, and the radius of the circular base is also 10.



The volume of the cylinder is _____ .

Answer 1000π

38 Find the volume of the following cone. (keep π in the answer)



Answer The volume is: $\frac{1}{3} \times (3^2 \times \pi \times 4) = 12\pi$

- 39 (1) If the volume of a cylinder is 375π and the height is 15π , the radius is _____ .
 (2) If the surface area of a cylinder is 128π , and the radius is 4, the height is _____ .

Answer 1:5

2:12

- 40 (1) The cylinder and the cone have the same base and the same volume. If the height of the cylinder is 3 cm, then the height of the cone is _____ cm.
 (2) The cylinder and the cone have the same volume and the same height. If the radius of the base of cylinder is 3 cm, then the radius of the base of cone is _____ cm.

Answer 1:9

2: $3\sqrt{3}$

41

Alex and Felicia each have cats as pets. Alex buys cat food in cylindrical cans that are 8 cm in diameter and 12 cm high. Felicia buys cat food in cylindrical cans that are 12 cm in diameter and 8 cm high. What is the ratio of the volume one of Alex's cans to the volume one of Felicia's cans?

Answer 2 : 3

42 A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased? (2004 AMC 12A Problem, Question #9)

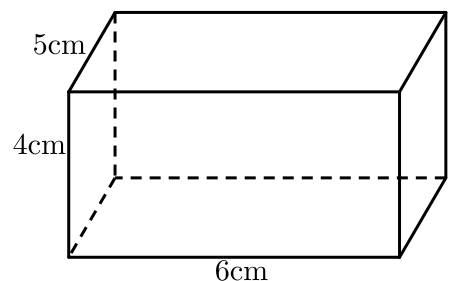
- A. 10 B. 25 C. 36 D. 50 E. 60

Answer C

Solution When the diameter is increased by 25%, it is increased by $\frac{5}{4}$, so the area of the base is increased by $\left(\frac{5}{4}\right)^2 = \frac{25}{16}$. To keep the volume the same, the height must be $\frac{1}{\frac{25}{16}} = \frac{16}{25}$ of the original height, which is a 36% reduction.

43 Find the volumes of the following solid figures:

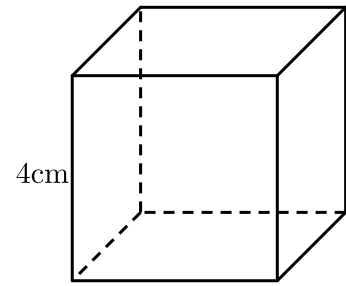
(1)



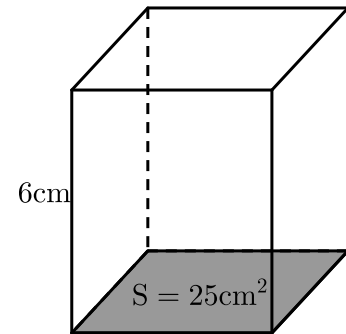
$V = \underline{\hspace{2cm}} \text{ cm}^3$.

(2)

(3) $V = \underline{\hspace{2cm}} \text{ cm}^3.$



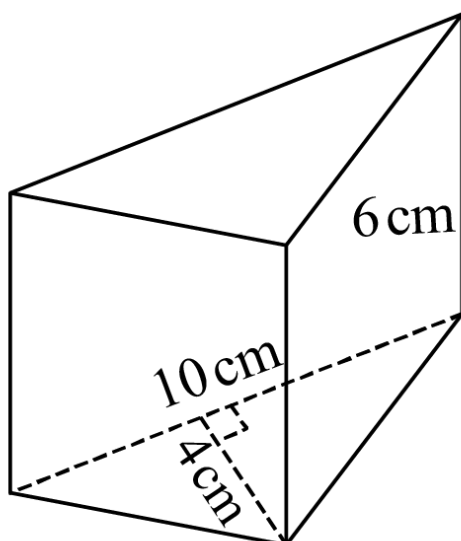
$V = \underline{\hspace{2cm}} \text{ cm}^3.$



- Answer**
- (1) 120
 - (2) 64
 - (3) 150

- Solution**
- (1) $6 \times 5 \times 4 = 120 \text{ (cm}^3\text{)} .$
 - (2) N/A
 - (3) N/A

44 Find the volume of the following right triangular prism:



Answer 120cm^3

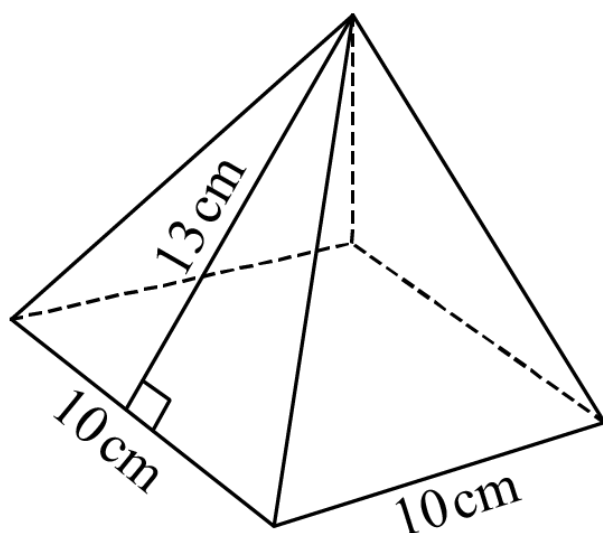
45 For the right square pyramid below:

The area of the base is _____ cm^2 .

The area of the four faces is _____ cm^2 .

The surface area is _____ cm^2 .

The volume is _____ cm^3 .



Answer 1:100

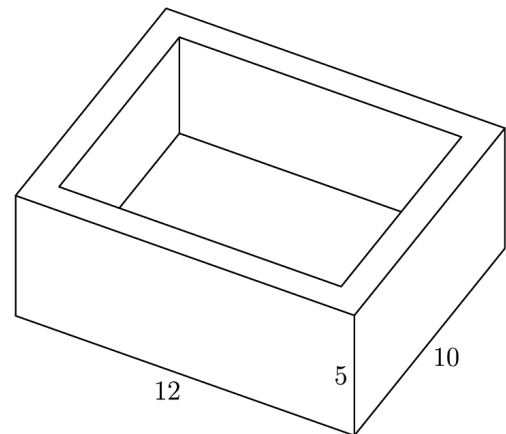
2:260

3:360

4:400

Solution N/A

- 46 Isabella uses one-foot cubical blocks to build a rectangular fort that is 12 feet long, 10 feet wide, and 5 feet high. The floor and the four walls are all one foot thick. How many blocks does the fort contain? () . (2018 AMC 8 Problems, Question #18)



- A. 204 B. 280 C. 320 D. 340 E. 600

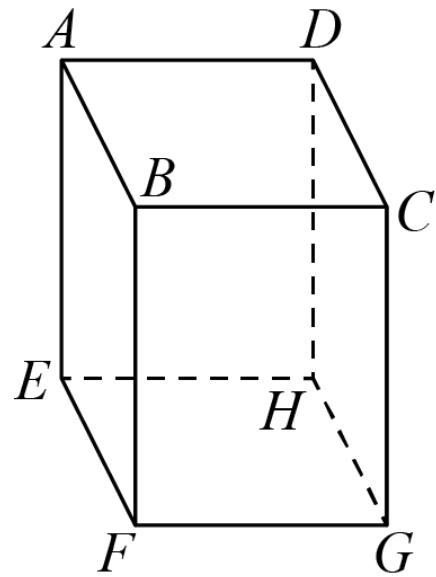
Answer B

Solution There are $10 \cdot 12 = 120$ cubes on the base of the box. Then, for each of the 4 layers above the bottom (as since each cube is 1 foot by 1 foot by 1 foot and the box is 5 feet tall, there are 4 feet left), there are $9 + 11 + 9 + 11 = 40$ cubes. Hence, the answer is $120 + 4 \cdot 40 = \boxed{(B)280}$.

We can just calculate the volume of the prism that was cut out of the original $12 \times 10 \times 5$ box. Each interior side of the fort will be 2 feet shorter than each side of the outside. Since the floor is 1 foot, the height will be 4 feet. So the volume of the interior box is $10 \times 8 \times 4 = 320\text{ft}^3$.

The volume of the original box is $12 \times 10 \times 5 = 600\text{ft}^3$. Therefore, the number of blocks contained in the fort is $600 - 320 = \boxed{(B)280}$.

- 47 In following rectangular prism, $AB = 4$, $BC = 3$, $BF = 6$. Find the volume of triangular prism $ABC - EFG$ and triangular pyramid $A - EFG$ and the length of AG .



Answer 36, 12, $\sqrt{61}$



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