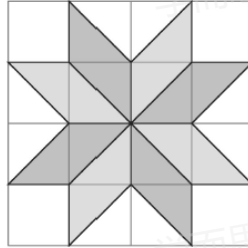


2025 AMC 8

- 1 The eight-pointed star, shown in the figure below, is a popular quilting pattern. What percent of the entire 4-by-4 grid is covered by the star?



- A. 40 B. 50 C. 60 D. 75 E. 80

Answer B

Solution **Total Grid Area:**

The grid is a 4×4 square, so its total area is $4 \times 4 = 16$.

Shaded Area:

The center square is fully shaded, contributing $1 \times 1 = 1$ square unit of area. There are 8 triangular regions around the center. Each triangle is half of a square, contributing $\frac{1}{2}$ square unit per triangle. The total area of these triangles is $8 \times \frac{1}{2} = 4$.

Percentage Covered:

$$\text{Percentage Covered} = \frac{\text{Shaded Area}}{\text{Total Area}} \times 100 = \frac{5}{16} \times 100 = 50\%.$$

2

The table below shows the ancient Egyptian hieroglyphs that were used to represent different numbers.

					
100,000	10,000	1,000	100	10	1

For example, the number 32 was represented by . What number was represented by the following combination of hieroglyphs?



A. 1, 423

B. 10, 423

C. 14, 023

D. 14, 203

E. 14, 230

Answer B

Solution $1 \times 10000 + 4 \times 100 + 2 \times 10 + 3 \times 1 = 10423$

3 *Buffalo Shuffle-o* is a card game in which all the cards are distributed evenly among all players at the start of the game. When Annika and 3 of her friends play *Buffalo Shuffle-o*, each player is dealt 15 cards. Suppose 2 more friends join the next game. How many cards will be dealt to each player?

A. 8

B. 9

C. 10

D. 11

E. 12

Answer C

Solution **Initial Game Setup:**

Annika and 3 of her friends, making a total of 4 players, are dealt 15 cards each. Therefore, the total number of cards is: $4 \times 15 = 60$.

Next Game Setup: If 2 more friends join, the total number of players becomes: $4 + 2 = 6$.

Cards Per Player: The cards are distributed evenly among all 6 players. Thus, each player receives: $\frac{60}{6} = 10$ cards.

Lucius is counting backward by 7s. His first three numbers are 100, 93, and 86. What is his 10th number?

- A. 30 B. 37 C. 42 D. 44 E. 47

Answer B

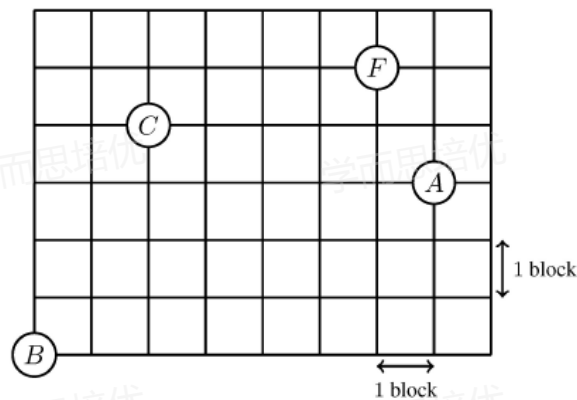
Solution **General Formula:**

The sequence can be expressed as: $a_n = 100 - 7(n - 1)$, where a_n represents the n -th number in the sequence.

Calculate a_{10} : Substitute $n = 10$ into the formula:

$$a_{10} = 100 - 7(10 - 1) = 100 - 7 \times 9 = 100 - 63 = 37.$$

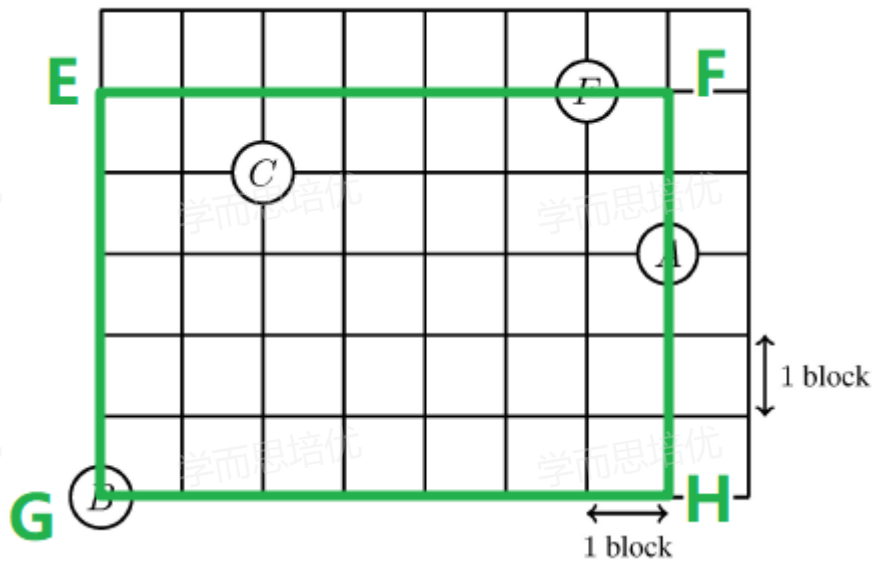
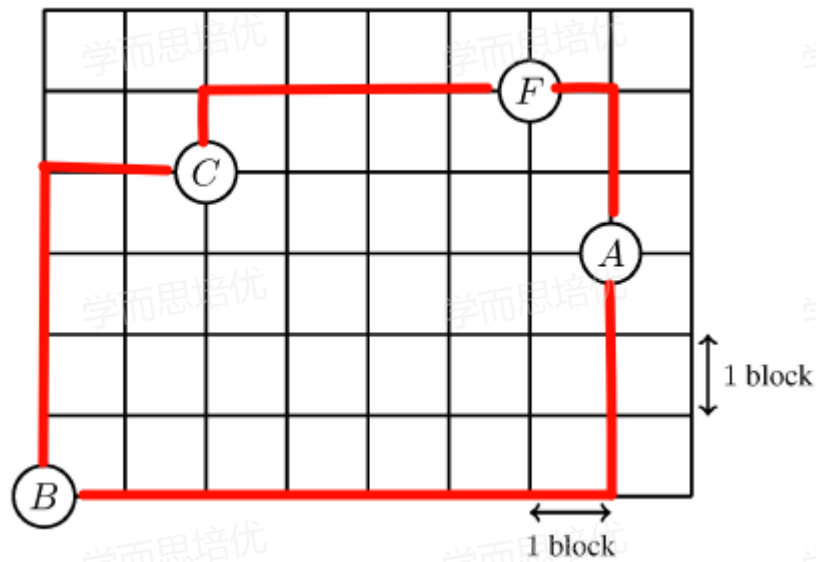
- 5 Betty drives a truck to deliver packages in a neighborhood whose street map is shown below. Betty starts at the factory (labeled F) and drives to location A , then B , then C , before returning to F . What is the shortest distance, in blocks, she can drive to complete the route?



- A. 20 B. 22 C. 24 D. 26 E. 28

Answer C

Solution



The shortest path is the perimeter of Rectangle $EFGH$.

$$P_{EFGH} = 2(7 + 5) = 24$$

- 6 Sekou writes the numbers 15, 16, 17, 18, 19. After he erases one of the numbers, the sum of the remaining four numbers is a multiple of 4. Which number did he erase?

A. 15 B. 16 C. 17 D. 18 E. 19

Answer C

Solution Sum of 15, 16, 17, 18, 19 is 85.

Missing 15: 70 isn't a multiple of 4.

Missing 16: 69 isn't a multiple of 4.

Missing 17: 68 is a multiple of 4.

Missing 18: 67 isn't a multiple of 4.

Missing 19: 66 isn't a multiple of 4.

7 On the most recent exam in Prof. Xochi's class,

5 students earned a score of at least 95%,

13 students earned a score of at least 90%,

27 students earned a score of at least 85%, and

50 students earned a score of at least 80%.

How many students earned a score of at least 80% and less than 90%?

A. 8

B. 14

C. 22

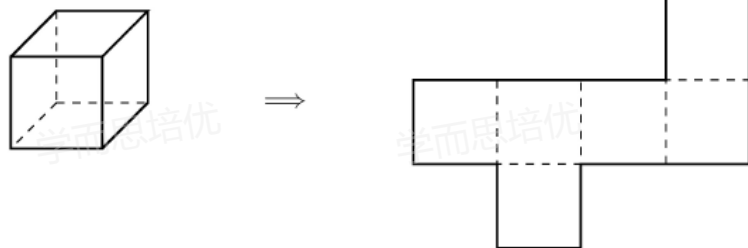
D. 37

E. 45

Answer D

Solution At least 80% and less than 90% = at least 80% – at least 90% = $50 - 13 = 37$

8 Isaiah cuts open a cardboard cube along some of its edges to form the flat shape shown on the right, which has an area of 18 square centimeters. What was the volume of the cube in cubic centimeters?



A. $3\sqrt{3}$

B. 6

C. 9

D. $6\sqrt{3}$

E. $9\sqrt{3}$

Answer A

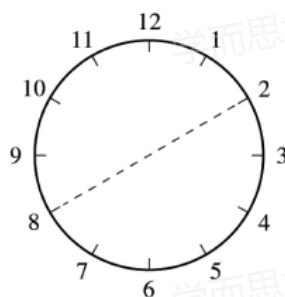
Solution Total area of 6 faces is 18cm^2 .

The area of one of the faces is $18 \div 6 = 3\text{cm}^2$.

Based on the area formula of the square, the side of one face is $\sqrt{3}\text{cm}$.

Then, $V = \text{bottom surface area} \times \text{side length} = 3\sqrt{3}$

- 9 Ningli looks at the 6 pairs of numbers directly across from each other on a clock. She takes the average of each pair of numbers. What is the average of the resulting 6 numbers?



- A. 5 B. 6.5 C. 8 D. 9.5 E. 12

Answer B

Solution **Solution 1:**

$$\text{Average of } (12, 6) = \frac{(12 + 6)}{2} = \frac{18}{2} = 9$$

$$\text{Average of } (1, 7) = \frac{(1 + 7)}{2} = \frac{8}{2} = 4$$

$$\text{Average of } (2, 8) = \frac{(2 + 8)}{2} = \frac{10}{2} = 5$$

$$\text{Average of } (3, 9) = \frac{(3 + 9)}{2} = \frac{12}{2} = 6$$

$$\text{Average of } (4, 10) = \frac{(4 + 10)}{2} = \frac{14}{2} = 7$$

$$\text{Average of } (5, 11) = \frac{(5 + 11)}{2} = \frac{16}{2} = 8$$

The average of these six numbers is:

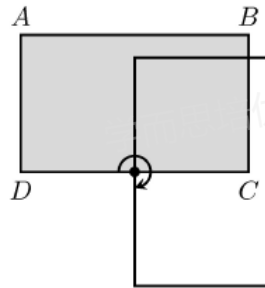
$$\text{Average} = \frac{9 + 4 + 5 + 6 + 7 + 8}{6} = \frac{39}{6} = 6.5$$

Solution 2:

Since we need to calculate the average of the averages of the six pairs, this can also be achieved by finding the average of the total sum of all the numbers.

$$\text{Average} = \frac{1 + 2 + 3 + \cdots + 10 + 11 + 12}{12} = \frac{78}{12} = 6.5$$

In the figure below, $ABCD$ is a rectangle with sides of length $AB = 5$ inches and $AD = 3$ inches. Rectangle $ABCD$ is rotated 90° clockwise around the midpoint of side $\overline{DC}$ to give a second rectangle. What is the total area, in square inches, covered by the two overlapping rectangles?



A. 21

B. 22.25

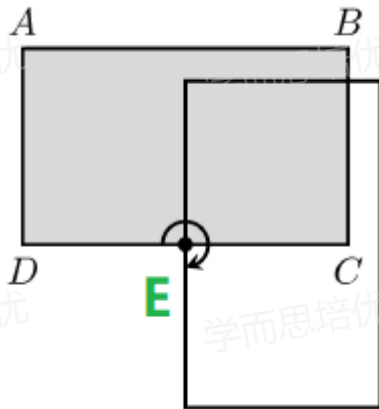
C. 23

D. 23.75

E. 25

Answer D

Solution



Total Area = Sum of both areas – Overlapping Area

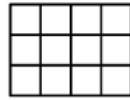
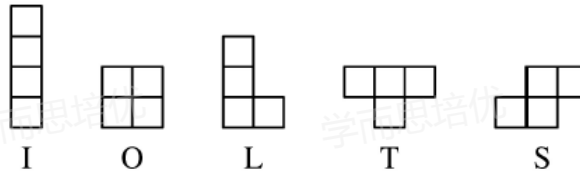
When rotated 90° clockwise around the midpoint of DC , the overlapping region forms a **square**.

The midpoint of DC divides the 5-inch side into two segments of 2.5 inches each.

The rotation causes the overlapping region to be a square with side length 2.5 inches (half of AB).

$$\text{Total area} = 2 \times \text{rectangle area} - \text{overlapped area} = 2 \times (5 \times 3) - \left(\frac{5}{2}\right)^2 = 30 - 2.5^2 = 23.75$$

A *tetromino* consists of four squares connected along their edges. There are five possible tetromino shapes, I, O, L, T, and S, shown below, which can be rotated or flipped over. Three tetrominoes are used to completely cover a 3×4 rectangle. At least one of the tiles is an S tile. What are the other two tiles?



A. I and L

B. I and T

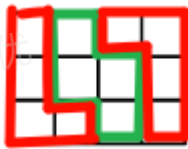
C. L and L

D. L and S

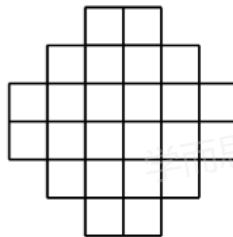
E. O and T

Answer C

Solution



- 12 The region shown below consists of 24 squares, each with side length 1 centimeter. What is the area, in square centimeters, of the largest circle that can fit inside the region, possibly touching the boundaries?



A. 3π

B. 4π

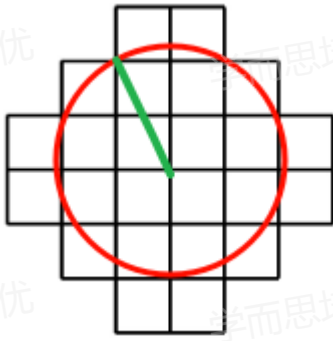
C. 5π

D. 6π

E. 8π

Answer C

Solution

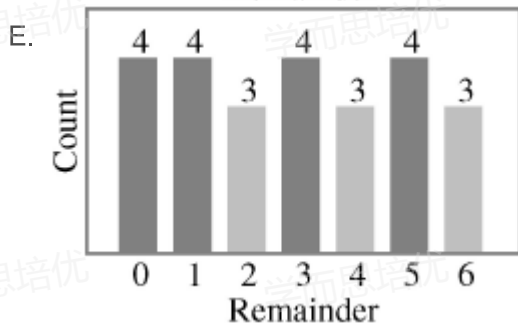
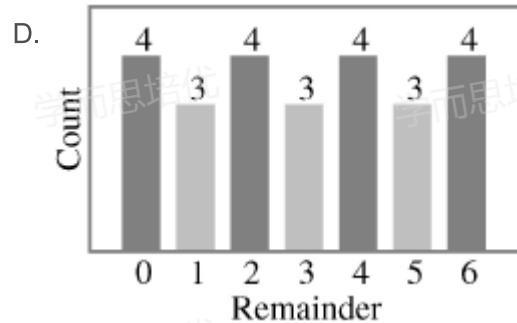
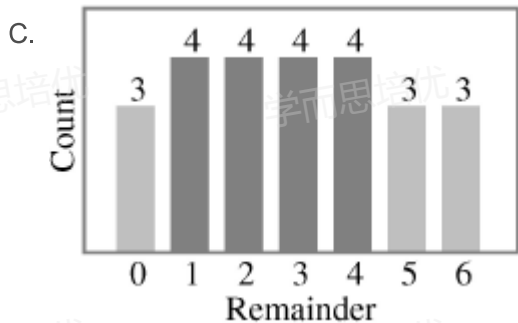
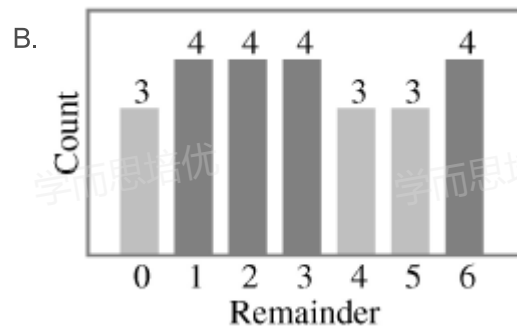
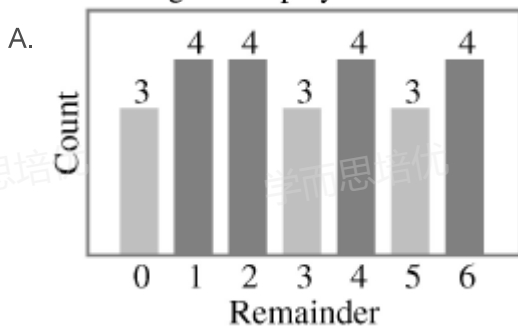


$$r = \sqrt{1^2 + 2^2} = \sqrt{5}$$

$$A = (\sqrt{5})^2 \pi = 5\pi$$

13

Each of the even numbers 2, 4, 6, ..., 50 is divided by 7. The remainders are recorded. Which histogram displays the number of times each remainder occurs?



Answer A

Solution To determine the histogram showing the frequency of remainders when each even number from 2 to 50 is divided by 7:

Total Numbers: The sequence $2, 4, 6, \dots, 50$ contains 25 even numbers.

Pattern of Remainders: Each term can be written as $2k$ ($k = 1, 2, \dots, 25$). When divided by 7, the remainders cycle every 7 terms:

Remainders for $k = 1$ to 7 : $2, 4, 6, 1, 3, 5, 0$

This cycle repeats every 7 values of k .

Full Cycles and Remaining Terms:

$25 \div 7 = 3$ full cycles with a remainder of 4.

Each full cycle contributes 3 **occurrences** for every remainder (0–6).

The remaining 4 terms follow the start of the cycle: $2, 4, 6, 1$, adding 1 **occurrence** to each of these remainders.

Final Counts:

Remainders $2, 4, 6, 1 : 3 + 1 = 4$ times each.

Remainders $0, 3, 5 : 3$ times each.

So answer is A

14

A number N is inserted into the list $2, 6, 7, 7, 28$. The mean is now twice as great as the median. What is N ?

A. 7

B. 14

C. 20

D. 28

E. 34

Answer E

Solution The median of 6 numbers is the average of the **3rd** and **4th** terms.

The mean must equal twice the median:

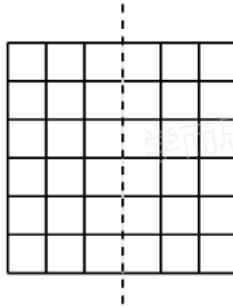
$$\frac{50 + N}{6} = 2 \times 7$$

$$\frac{50 + N}{6} = 14$$

$$N = 34$$

15

Kei draws a 6-by-6 grid. He colors 13 of the unit squares silver and the remaining squares gold. Kei then folds the grid in half vertically, forming pairs of overlapping unit squares. Let m and M equal the least and greatest possible number of gold-on-gold pairs, respectively. What is the value of $m + M$?



A. 12

B. 14

C. 16

D. 18

E. 20

Answer C

Solution There are $6 \times 6 = 36$ grids in total. Then, the number of golden grid is $36 - 13 = 23$. All 23 golden grid can be used to constituted gold-on-gold pair. So we can get the greatest possible number of gold-on-gold pair by evenly splitting all golden grid into two sides. That is $23 \div 2 = 11R1$. For the same reason, we can get the least possible number of gold-on-gold pair by putting all golden grid in one side. That is $23 - 36 \div 2 = 5$
 $m + M = 5 + 11 = 16$

16 Five distinct integers from 1 to 10 are chosen, and five distinct integers from 11 to 20 are chosen. No two numbers differ by exactly 10. What is the sum of the ten chosen numbers?

A. 95

B. 100

C. 105

D. 110

E. 115

Answer C

Solution For each number k in 1–10, its counterpart in 11–20 is $k + 10$.

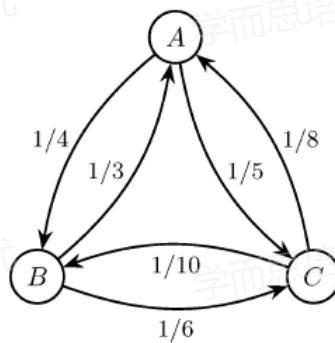
If k is chosen from 1–10, $k + 10$ cannot be chosen from 11–20, and vice versa.

Each valid pair $(k, 21 - k)$ sums to 21 (e.g., $1 + 20 = 21, 2 + 19 = 21$, etc.).

With 5 such pairs, the total sum is:

$$21 \times 5 = 105$$

- 17 In the land of Markovia, there are three cities: A , B , and C . There are 100 people who live in A , 120 who live in B , and 160 who live in C . Everyone works in one of the three cities, and a person may work in the same city where they live. In the figure below, an arrow pointing from one city to another is labeled with the fraction of people living in the first city who work in the second city. (For example, $\frac{1}{4}$ of the people who live in A work in B .) How many people work in A ?



- A. 55 B. 60 C. 85 D. 115 E. 160

Answer D

Solution Residents of A working in A :

$$\text{Fraction remaining in } A : 1 - \frac{1}{4} - \frac{1}{5} = \frac{11}{20}.$$

Calculation:

$$\frac{11}{20} \times 100 = 55 \text{ people}$$

Residents of B working in A :

$$\text{Fraction working in } A : \frac{1}{3}.$$

Calculation:

$$\frac{1}{3} \times 120 = 40 \text{ people}$$

Residents of C working in A :

$$\text{Fraction working in } A : \frac{1}{8}.$$

Calculation:

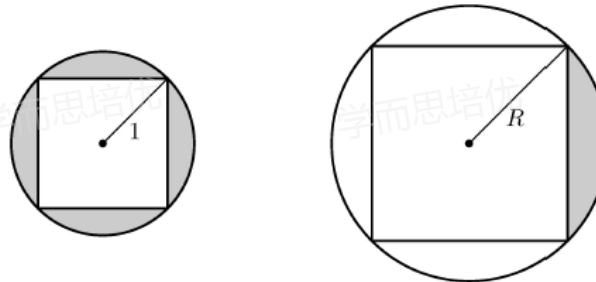
$$\frac{1}{8} \times 160 = 20 \text{ people.}$$

Total People Working in A :

$$55 + 40 + 20 = 115$$

18

The circle shown below on the left has a radius of 1 unit. The region between the circle and the inscribed square is shaded. In the circle shown on the right, one quarter of the region between the circle and the inscribed square is shaded. The shaded regions in the two circles have the same area. What is the radius R , in units, of the circle on the right?



A. $\sqrt{2}$

B. 2

C. $2\sqrt{2}$

D. 4

E. $4\sqrt{2}$

Answer B

Solution **Solution by Pythagorean Theorem:**

Left Circle:

The radius of the circle is 1, so the area of the circle is: $A_{\text{left}} = \pi(1)^2 = \pi$.

The square is inscribed in the circle, meaning its diagonal equals the diameter of the circle, which is 2. Let the side length of the square be s . Using the Pythagorean Theorem:

$$s^2 + s^2 = 2^2$$

$$2s^2 = 4$$

$$s^2 = 2$$

$$s = \sqrt{2}.$$

The area of the square is: $A_{\text{square}} = s^2 = 2$. The shaded area is the difference between the circle's area and the square's area: $A_{\text{shaded}} = A_{\text{circle}} - A_{\text{square}} = \pi - 2$.

Right Circle:

The diagonal of the square is equal to the diameter of the circle, so the diameter is $2R$. Let the side length of the square be t . Using the Pythagorean Theorem: $t^2 + t^2 = (2R)^2$

$$2t^2 = 4R^2$$

$$t^2 = 2R^2.$$

The area of the square is: $A_{\text{square}} = t^2 = 2R^2$. The area of the circle is: $A_{\text{circle}} = \pi R^2$. The total shaded area is one-quarter of the region between the circle and the square:

$$A_{\text{shaded}} = \frac{1}{4}(A_{\text{circle}} - A_{\text{square}}) = \frac{1}{4}(\pi R^2 - 2R^2) = \frac{1}{4}R^2(\pi - 2).$$

The shaded areas in the two circles are equal: $\pi - 2 = \frac{1}{4}R^2(\pi - 2)$.

Divide both sides by $\pi - 2$ (since $\pi - 2 > 0$): $1 = \frac{1}{4}R^2$

$$R^2 = 4$$

$$R = 2.$$

Advanced Solution:

Shaded area in left circle is $\pi - 2$. Since the shaded regions in two circles have the same

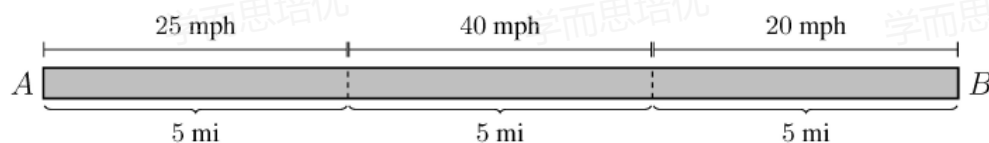
area, the relation between two circle can be expressed as $\frac{\pi}{\pi - 2} = \frac{\pi R^2}{4(\pi - 2)}$

$$1 = \frac{R^2}{4}$$

$$R = 2$$

19

Two towns, A and B , are connected by a straight road, 15 miles long. Traveling from town A to town B , the speed limit changes every 5 miles: from 25 to 40 to 20 miles per hour (mph). Two cars, one at town A and one at town B , start moving toward each other at the same time. They drive at exactly the speed limit in each portion of the road. How far from town A , in miles, will the two cars meet?



A. 7.75

B. 8

C. 8.25

D. 8.5

E. 8.75

Answer D

Solution **Solution 1:**

Distance = Speed $\times$ Time.

Let x be the distance traveled by the car starting from A , and y be the distance traveled by the car starting from B . The total distance between A and B is: $x + y = 15$.

Car from A :

First segment (5 miles at 25 mph): $t_1 = \frac{5}{25} = 0.2$ hours.

Second segment ($x - 5$ miles at 40 mph): $t_2 = \frac{x - 5}{40}$.

The total time for the car from A is: $t_A = t_1 + t_2 = 0.2 + \frac{x-5}{40}$.

Car from B :

First segment (5 miles at 20 mph):

$$t_3 = \frac{5}{20} = 0.25 \text{ hours.}$$

Second segment ($y - 5$ miles at 40 mph): $t_4 = \frac{y-5}{40}$.

The total time for the car from B is: $t_B = t_3 + t_4 = 0.25 + \frac{y-5}{40}$.

Write equation:

Since the two cars meet at the same time: $t_A = t_B$.

Substitute $y = 15 - x$ into the equation: $0.2 + \frac{x-5}{40} = 0.25 + \frac{15-x-5}{40}$.

Simplify: $0.2 + \frac{x-5}{40} = 0.25 + \frac{10-x}{40}$.

Combine terms: $\frac{x-5}{40} + \frac{x}{40} = 0.25 - 0.2 + \frac{10}{40}$.

Simplify further: $\frac{2x-5}{40} = 0.05 + \frac{10}{40}$.

Convert 0.05 to a fraction: $\frac{2x-5}{40} = \frac{2}{40} + \frac{10}{40}$.

Combine the right-hand side: $\frac{2x-5}{40} = \frac{12}{40}$.

Solve for x : $2x - 5 = 12 \implies 2x = 17 \implies x = 8.5$.

Solution 2:

Car A takes $\frac{5}{25} = 0.2$ hours to finish the first segment from Town A while Car B takes

$\frac{5}{20} = 0.25$ hours to finish the first segment from Town B. Therefore, Car A will enter the

second segment $0.25 - 0.2 = 0.05$ hours earlier than Car B, which is $0.05 \times 40 = 2$ miles.

Then, Car A and Car B will move toward each other by $\frac{5-2}{2} = 1.5$ miles until they meet. So

the distance between meeting point and Town A is $5 + 2 + 1.5 = 8.5$ miles.

20

Sarika, Dev, and Rajiv are sharing a large block of cheese. They take turns cutting off half of what remains and eating it: first Sarika eats half of the cheese, then Dev eats half of the remaining half, then Rajiv eats half of what remains, then back to Sarika, and so on. They stop when the cheese is too small to see. About what fraction of the original block of cheese does Sarika eat in total?

A. $\frac{4}{7}$

B. $\frac{3}{5}$

C. $\frac{2}{3}$

D. $\frac{3}{4}$

E. $\frac{7}{8}$

Answer A

Solution

Regular Solution:

The cheese ratio of each person is always **Sarika : Dev : Rajiv** = 4 : 2 : 1. So the fraction of cheese Sarika eats in total is $\frac{4}{4+2+1} = \frac{4}{7}$

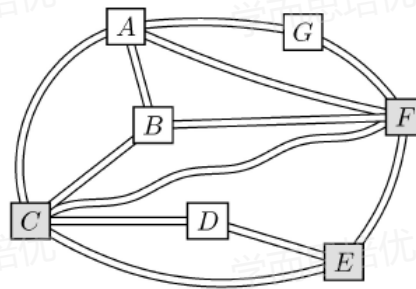
Advanced solution:

We will use the sum of infinite geometric sequence formula to solve.

$S = \frac{a}{1-r}$, where a is the first element and r is the ratio.

$$S_{\text{Sarika}} = \frac{a}{1-r} = \frac{\frac{1}{2}}{1-\frac{1}{8}} = \frac{4}{7}$$

- 21 The Königsberg School has assigned grades 1 through 7 to pods *A* through *G*, one grade per pod. Some of the pods are connected by walkways, as shown in the figure below. The school noticed that each pair of connected pods has been assigned grades differing by 2 or more grade levels. (For example, grades 1 and 2 will not be in pods directly connected by a walkway.) What is the sum of the grade levels assigned to pods *C*, *E*, and *F*?



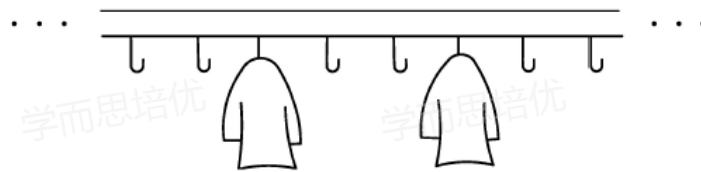
- A. 12 B. 13 C. 14 D. 15 E. 16

Answer A

Solution Since *C* and *F* both have 5 connected walkways, according to the description, *C* and *F* have to be Grade 1 and Grade 7. Then, *E* has to be one of the Grade 3, Grade 4, or Grade 5. By enumeration, we can find the *E* is Grade 4. $C + E + F = 1 + 4 + 7 = 12$

22

A classroom has a row of 35 coat hooks. Paulina likes coats to be equally spaced, so that there is the same number of empty hooks before the first coat, after the last coat, and between every coat and the next one. Suppose there is at least 1 coat and at least 1 empty hook. How many different numbers of coats can satisfy Paulina's pattern?



- A. 2 B. 4 C. 5 D. 7 E. 9

Answer D

Solution Suppose that the number of coats is $= n$ and space between each coat is $= n + 1$. Then the number of empty hooks can be expressed as $35 - n$. So we need to find all possible n that can satisfy $\frac{35 - n}{n + 1}$ is a integer. There are seven possible n that satisfy this constraint, $n = 1, 2, 3, 5, 8, 11, 17$

23 How many four-digit numbers have all three of the following properties?

- (I) The tens digit and ones digit are both 9.
- (II) The number is 1 less than a perfect square.
- (III) The number is the product of exactly two prime numbers.

- A. 0 B. 1 C. 2 D. 3 E. 4

Answer B

Solution **1. Tens and Ones Digits:** The tens and ones digits are both 9, so the number has the form $\overline{ab99}$.

2. 1 Less Than a Perfect Square: The number can be expressed as $n^2 - 1$ for some integer n .

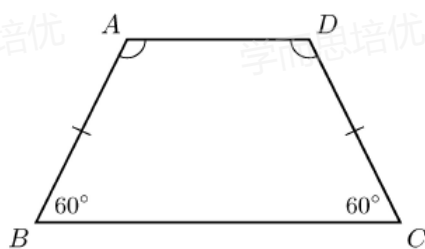
3. Product of Exactly Two Prime Numbers: The number should be a semiprime, which means it is the product of exactly two prime factors.

From the first property, the number has the form $100a + 10b + 99$. Since the number is $n^2 - 1$, we need n^2 to be a four-digit number that ends in 00 (so that $n^2 - 1$ ends in 99).

From the second property, because $n^2 - 1$ can be factored as $(n - 1)(n + 1)$, the number must be expressible as the product of two consecutive integers. One of which is $n - 1$ and the other $n + 1$. From the third property, $n - 1$ and $n + 1$ must both be primes.

All the computational constraints narrow down the search to specific ranges: $\sqrt{1000} \approx 31.6$ to $\sqrt{9999} \approx 99.99$. That is range $\{40, 50, 60, 70, 80, 90\}$. By calculation, only 60 fits our requirement.

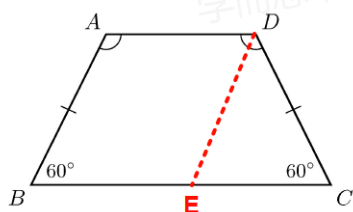
- 24 In trapezoid $ABCD$, angles B and C measure 60° and $AB = DC$. The side lengths are all positive integers and the perimeter of $ABCD$ is 30 units. How many non-congruent trapezoids satisfy all of these conditions?



- A. 0 B. 1 C. 2 D. 3 E. 4

Answer E

Solution Solution 1 by cutting:



Draw a segment from D to E which is parallel to AB . Suppose $CD = x$, $AD = y$. Then

$BC = BE + EC = AD + CD = x + y$. The perimeter can be written as

$$AD + DC + CB + BA = y + x + (x + y) + x = 2y + 3x = 30.$$

$$y = \frac{30 - 3x}{2}$$

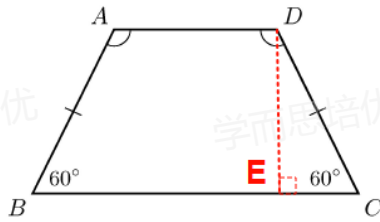
$$x = 2, y = 12$$

$$x = 4, y = 9$$

$$x = 6, y = 6$$

$$x = 8, y = 3$$

Solution 2 by trigonometric function:



Suppose $AB = CD = x$, $AD = y$, then $CE = \frac{x}{2}$, $BC = AD + 2(\frac{x}{2})$. Therefore, the

perimeter can be written as $AD + DC + CB + BA = y + x + (y + x) + x = 2y + 3x = 30$.

$$y = \frac{30 - 3x}{2}$$

$$x = 2, y = 12$$

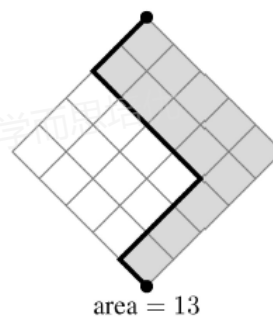
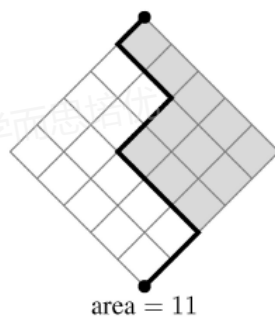
$$x = 4, y = 9$$

$$x = 6, y = 6$$

$$x = 8, y = 3$$

25

Makayla finds all the possible ways to draw a path in a 5×5 diamond-shaped grid. Each path starts at the bottom of the grid and ends at the top, always moving one unit northeast or northwest. She computes the area of the region between each path and the right side of the grid. Two examples are shown in the figures below. What is the sum of the areas determined by all possible paths?



A. 2520

B. 3150

C. 3840

D. 4730

E. 5050

Answer B

Solution Solution 1 by combination:

Number of Paths:

On this grid, each path consists of 10 moves, either northeast (NE) or northwest (NW), starting from the bottom corner and ending at the top corner. The total number of paths equals the number of ways to choose 10 moves that include a certain number of NE and NW moves. Since you always move from the bottom to the top, the number of NE moves must equal the number of NW moves. That is anytime you choose move NE for once, then you have to move to NW for once later. Because of that, you actually only have five chance to choose direction. So **number of path** = $\binom{10}{5} = 252$.

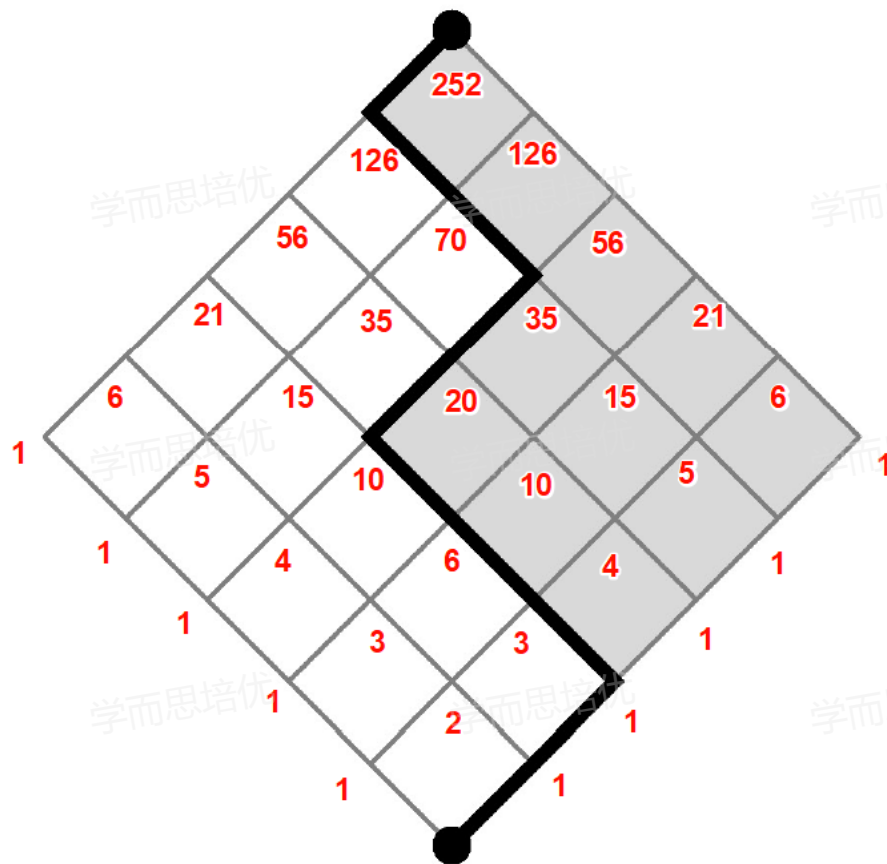
Average Area:

Since the amount of grid at left side always equal to the amount of grid at right side. So **average area** = $5^2 \div 2 = 12.5$

Total Area:

Finally, **sum of area** = $252 \times 12.5 = 3150$

Solution 2 by enumeration:



The number on the node(intersection) represents the number of different ways to reach.

Average Area:

Since the amount of grid at left side always equal to the amount of grid at right side, So

$$\text{average area} = 5^2 \div 2 = 12.5$$

Total Area:

$$\text{Finally, sum of area} = 252 \times 12.5 = 3150$$

Solution 2 by enumeration: