



MAA
AMC AMERICAN
MATHEMATICS
COMPETITION

MAA American Mathematics Competitions

26th Annual

AMC 10 A

Wednesday, November 6, 2024

INSTRUCTIONS

1. DO NOT TURN TO THE NEXT PAGE UNTIL YOUR COMPETITION MANAGER TELLS YOU TO BEGIN.
2. This is a 25-question multiple-choice competition. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check blackened answers for accuracy and erase errors completely. Only answers that are properly marked on the answer sheet will be scored.
4. **SCORING:** You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only blank scratch paper, rulers, compasses, and erasers are allowed as aids. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the competition will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. You will have 75 minutes to complete the competition once your competition manager tells you to begin.
8. When you finish with the competition, please follow the directions of your competition manager.

The problems and solutions for this AMC 10 A were prepared
by the MAA AMC 10/12 Editorial Board under the direction of
Gary Gordon and Carl Yerger, co-Editors-in-Chief.

The MAA AMC office reserves the right to disqualify scores from a school if it determines that the rules or the required security procedures were not followed.

The publication, reproduction, or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

Students who score well on this AMC 10 will be invited to take the 43rd annual American Invitational Mathematics Examination (AIME) on Thursday, February 6, 2025, or Wednesday, February 12, 2025. More details about the AIME can be found at maa.org/AMC.

A

1. What is the value of $101 \cdot 9,901 - 99 \cdot 10,101$?

- (A) 2 (B) 20 (C) 21 (D) 200 (E) 2020

$$\begin{aligned}
 101 \cdot 9901 - 99 \cdot 10101 &= (100+1)(9900+1) - (100-1)(10100+1) \\
 &= 100 \cdot 9900 + 10000 + 1 - 100 \cdot 10100 + 10000 + 1 \\
 &= -20000 + 20000 + 2 \\
 &= 2
 \end{aligned}$$

B

2. A model used to estimate the time it will take to hike to the top of a mountain on a trail is of the form $T = aL + bG$, where a and b are constants, T is the time in minutes, L is the length of the trail in miles, and G is the altitude gain in feet. The model estimates that it will take 69 minutes to hike to the top if a trail is 1.5 miles long and ascends 800 feet, as well as if a trail is 1.2 miles long and ascends 1100 feet. How many minutes does the model estimate it will take to hike to the top if the trail is 4.2 miles long and ascends 4000 feet?

- (A) 240 (B) 246 (C) 252 (D) 258 (E) 264

$$\begin{aligned}
 \begin{cases} 69 = 1.5a + 800b \\ 69 = 1.2a + 1100b \end{cases} \\
 \Rightarrow 0.3a = 300b \\
 a = 1000b
 \end{aligned}$$

$$\begin{aligned}
 \text{Ans} &= 4.2 \times 30 + 4000 \times \frac{3}{100} \\
 &= 126 + 120 \\
 &= 246
 \end{aligned}$$

$$\begin{aligned}
 \Rightarrow 69 &= 1500b + 800b \\
 \begin{cases} b = \frac{69}{2300} = \frac{3}{100} \\ a = 30 \end{cases}
 \end{aligned}$$

B

3. Let n be the least prime number that can be written as the sum of 5 distinct prime numbers. What is the sum of the digits of n ?

- (A) 5 (B) 7 (C) 8 (D) 10 (E) 11

Can't have 2 because sum is even.

$$\textcircled{1} \text{ Try } 3+5+7+11+13 = 39$$

$$\begin{aligned}
 \textcircled{2} \text{ Try } 3+5+7+11+17 &= 43 \\
 \Rightarrow 4+3 &= 7
 \end{aligned}$$

4. The number 2024 is written as the sum of not necessarily distinct two-digit numbers. What is the least number of two-digit numbers needed to write this sum?

(A) 20 (B) 21 (C) 22 (D) 23 (E) 24

Want $2024 = \underbrace{99 + 99 + \dots + 99}_n + x$

$$\Rightarrow n = \left\lfloor \frac{2024}{99} \right\rfloor = 20$$

Need $20 + 1 = 21$ numbers

5. What is the least value of n such that $n!$ is a multiple of 2024?

(A) 11 (B) 21 (C) 22 (D) 23 (E) 253

$$2024 = 2^3 \times 11 \times 23 \Rightarrow 23 \mid n! \Rightarrow \text{Ans is } 23$$

6. What is the minimum number of successive swaps of adjacent letters in the string ABCDEF that are needed to change the string to FEDCBA? (For example, 3 swaps are required to change ABC to CBA; one such sequence of swaps is $ABC \rightarrow BAC \rightarrow BCA \rightarrow CBA$.)

(A) 6 (B) 10 (C) 12 (D) 15 (E) 24

5 steps: BCDEFA

4 steps: CDEFBA

Ans = 15

3 steps: DEFCBA

2 steps: EFDCBA

1 step: FEDCBA

7. The product of three integers is 60. What is the least possible positive sum of the three integers?

(A) 2 (B) 3 (C) 5 (D) 6 (E) 13

Least positive sum: 2 Negative + 1 Positive

$$60 = 2^2 \times 3 \times 5 \Rightarrow 10, -6, -1$$

$$\Rightarrow \text{Sum} = 3$$

8. Amy, Bomani, Charlie, and Daria work in a chocolate factory. On Monday Amy, Bomani, and Charlie started working at 1:00 PM and were able to pack 4, 3, and 3 packages, respectively, every 3 minutes. At some later time, Daria joined the group, and Daria was able to pack 5 packages every 4 minutes. Together, they finished packing 450 packages at exactly 2:45 PM. At what time did Daria join the group?

(A) 1:25 PM (B) 1:35 PM (C) 1:45 PM (D) 1:55 PM (E) 2:05 PM

Before: 10 packs every 3 mins

$$\frac{10}{3}x + \frac{55}{12}(105 - x) = 450$$

After: 55 packs every 12 mins

$$8x + 11 \cdot 105 - 11x = 1080$$

$$3x = 75$$

$$x = 25$$

Total time: 105 mins

Let x be the time before Daria

$$\Rightarrow 1:25$$

- B. In how many ways can 6 juniors and 6 seniors form 3 disjoint teams of 4 people so that each team has 2 juniors and 2 seniors?

(A) 720 (B) 1350 (C) 2700 (D) 3280 (E) 8100

$$\frac{(6C_2)^2 \cdot (4C_2)^2}{3!} = 1350$$

- C. 10. Consider the following operation. Given a positive integer n , if n is a multiple of 3, then you replace n by $\frac{n}{3}$. If n is not a multiple of 3, then you replace n by $n+10$. Then continue this process. For example, beginning with $n = 4$, this procedure gives $4 \rightarrow 14 \rightarrow 24 \rightarrow 8 \rightarrow 18 \rightarrow 6 \rightarrow 2 \rightarrow 12 \rightarrow \dots$. Suppose you start with $n = 100$. What value results if you perform this operation exactly 100 times?

(A) 10 (B) 20 (C) 30 (D) 40 (E) 50

$$100 \rightarrow 110 \rightarrow 120 \rightarrow 40 \rightarrow 50 \rightarrow 60 \rightarrow 20 \rightarrow 30 \rightarrow 10 \rightarrow 20$$

5 Times ① ② ③

After 5 times, cycle through 20, 30, 10

$$\Rightarrow 100 - 5 = 95 \equiv 2 \pmod{3} \Rightarrow \boxed{30}$$

- D. 11. How many ordered pairs of integers (m, n) satisfy $\sqrt{n^2 - 49} = m$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) infinitely many

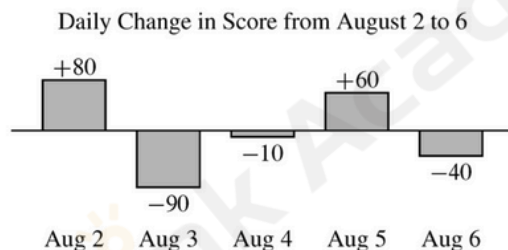
$$n^2 - 49 = m^2 \quad (n^2 \geq 49, m \geq 0)$$

$$n^2 - m^2 = 49$$

$$(n+m)(n-m) = 49 = 7^2$$

$$\begin{array}{llll} \textcircled{1} \begin{cases} n+m = n-m = 7 \\ \Rightarrow \begin{cases} n=7 \\ m=0 \end{cases} \end{cases} & \textcircled{2} \begin{cases} n+m = 49, n-m=1 \\ \Rightarrow \begin{cases} n=25 \\ m=24 \end{cases} \end{cases} & \textcircled{3} \begin{cases} n+m = -7, n-m = -7 \\ \Rightarrow \begin{cases} n=-7 \\ m=0 \end{cases} \end{cases} & \textcircled{4} \begin{cases} n+m = -1, n-m = -49 \\ \Rightarrow \begin{cases} n=-25 \\ m=24 \end{cases} \end{cases} \end{array}$$

- E. 12. Zelda played the Adventures of Math game on August 1 and scored 1700 points. She continued to play daily over the next 5 days. The bar chart below shows the daily change in her score compared to the day before. (For example, Zelda's score on August 2 was $1700 + 80 = 1780$ points.) What was Zelda's average score in points over the 6 days?



Daily Change compared to 1700:

$$\begin{array}{r} +80 \\ -10 \\ -20 \\ +40 \\ +0 \\ \hline = 90 \end{array}$$

Average change:

$$90 \div 6 = 15$$

$$\Rightarrow \boxed{1715}$$

(A) 1700 (B) 1702 (C) 1703 (D) 1713 (E) 1715

- C 13. Two transformations are said to *commute* if applying the first followed by the second gives the same result as applying the second followed by the first. Consider these four transformations of the coordinate plane:

- a translation 2 units to the right,
- a 90° -rotation counterclockwise about the origin,
- a reflection across the x -axis, and
- a dilation centered at the origin with scale factor 2.

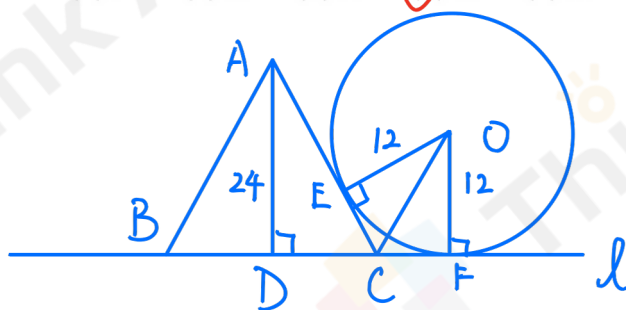
Of the 6 pairs of distinct transformations from this list, how many commute?

- (A) 1 (B) 2 ☒ (C) 3 (D) 4 (E) 5

For (x, y) , $T_1: (x, y) \rightarrow (x+2, y)$ Commute; T_1, T_3
 $T_2: (x, y) \rightarrow (-y, x)$ T_2, T_4
 $T_3: (x, y) \rightarrow (x, -y)$ T_3, T_4
 $T_4: (x, y) \rightarrow (2x, 2y)$

- D 14. One side of an equilateral triangle of height 24 lies on line ℓ . A circle of radius 12 is tangent to ℓ and is externally tangent to the triangle. The area of the region exterior to the triangle and the circle and bounded by the triangle, the circle, and line ℓ can be written as $a\sqrt{b} - c\pi$, where a, b , and c are positive integers and b is not divisible by the square of any prime. What is $a + b + c$?

- (A) 72 (B) 73 (C) 74 ☒ (D) 75 (E) 76



$m\angle EOF = m\angle ACB = 60^\circ$
 $\Rightarrow m\angle COF = 30^\circ, CF = 4\sqrt{3}$

Area = $2A_{\triangle COF} - A_{\text{sector}}$
 $= 12 \cdot 4\sqrt{3} - \frac{1}{6}\pi \cdot 12^2$
 $= 48\sqrt{3} - 24\pi$

Ans = $48 + 3 + 24 = \boxed{75}$

- E 15. Let M be the greatest integer such that both $M + 1213$ and $M + 3773$ are perfect squares. What is the units digit of M ?

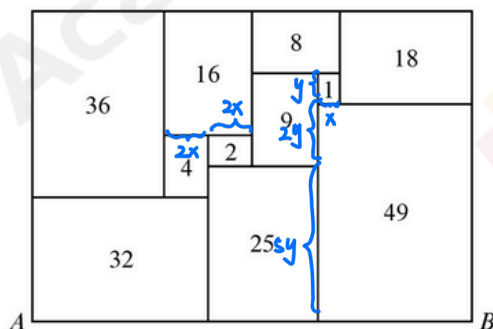
- (A) 1 (B) 2 (C) 3 (D) 6 ☒ (E) 8

$\begin{cases} M + 1213 = a^2 \\ M + 3773 = b^2 \end{cases}$
 $\Rightarrow b^2 - a^2 = 3773 - 1213$
 $(b+a)(b-a) = 2560 = 16^2 \times 10$

M is greatest, maximize $a+b$
 $\Rightarrow$ If $b-a=1$, a^2+b^2 must be odd, contradict
 $\Rightarrow \begin{cases} b-a=2 \\ b+a=1280 \end{cases} \Rightarrow \begin{cases} a=639 \\ b=641 \end{cases}$

$\Rightarrow M = 641^2 - 3773$
 $= 4071108$ (Ans = $\boxed{8}$)

- D 16. All of the rectangles in the figure below, which is drawn to scale, are similar to the enclosing rectangle. Each number represents the area of its rectangle. What is length AB ?



- (A) $4 + 4\sqrt{5}$ (B) $10\sqrt{2}$ (C) $5 + 5\sqrt{5}$ (D) $10\sqrt[4]{8}$ (E) 20

Area 32:

$$l \cdot \frac{l}{\sqrt{2}} = 32$$

$$l = 4\sqrt[4]{8}$$

Area 25:

$$w \cdot \sqrt{2}w = 25$$

$$w = \frac{5}{\sqrt[4]{2}}$$

$$= \frac{5}{2}\sqrt[4]{8}$$

Area 49:

$$w \cdot \sqrt{2}w = 49$$

$$w = \frac{7}{2}\sqrt[4]{8}$$

$$AB = 10\sqrt[4]{8}$$

- E 17. Two teams are in a best-two-out-of-three playoff: the teams will play at most 3 games, and the winner of the playoff is the first team to win 2 games. The first game is played on Team A's home field, and the remaining games are played on Team B's home field. Team A has a $\frac{2}{3}$ chance of winning at home, and its probability of winning when playing away from home is p . Outcomes of the games are independent. The probability that Team A wins the playoff is $\frac{1}{2}$. Then p can be written in the form $\frac{1}{2}(m - \sqrt{n})$, where m and n are positive integers. What is $m + n$?

- (A) 10 (B) 11 (C) 12 (D) 13 (E) 14

$$P(WW) = \frac{2}{3}P$$

$$\Rightarrow \frac{2}{3}P + \frac{2}{3}(1-P)P + \frac{1}{3}P^2 = \frac{1}{2}$$

$$P(WLW) = \frac{2}{3}(1-P)P$$

$$4P + 4P - 4P^2 + 2P^2 = 3$$

$$P(LWW) = \frac{1}{3}P^2$$

$$2P^2 - 8P + 3 = 0$$

$$P = \frac{8 \pm \sqrt{64 - 24}}{4}$$

$$= \frac{4 \pm \sqrt{10}}{2} = \frac{1}{2}(4 - \sqrt{10})$$

$$\text{Ans} = 14$$

18. There are exactly K positive integers b with $5 \leq b \leq 2024$ such that the base- b integer 2024_b is divisible by 16 (where 16 is in base ten). What is the sum of the digits of K ?

(A) 16 (B) 17 (C) 18 (D) 20 (E) 21

$$2024_b = 2b^3 + 2b + 4 \Rightarrow \text{We want: } \underset{7}{b^3} + \underset{7}{b} + \underset{2}{2} \equiv 0 \pmod{8}$$

$$= 2(b^3 + b + 2)$$

$$\Rightarrow \text{When } b \equiv 3, 6 \text{ or } 7 \pmod{8}, b^3 + b + 2 \equiv 0 \pmod{8}$$

There are $\lfloor \frac{2024-8}{8} \rfloor = 252$ cycles of remainder, and number 6 and 7.

$$\text{Ans} = 3 \times 252 + 2 = 758 \Rightarrow \boxed{20}$$

19. The first three terms of a geometric sequence are the integers a , 720, and b , where $a < 720 < b$. What is the sum of the digits of the least possible value of b ?

(A) 9 (B) 12 (C) 16 (D) 18 (E) 21

Suppose common ratio is $\frac{p}{q}$

$$\Rightarrow \begin{cases} a = 720 \div (\frac{p}{q}) \\ b = 720 \cdot (\frac{p}{q}) \end{cases} \Rightarrow \begin{cases} p > q \\ p, q \mid 720 \\ \text{minimize } \frac{p}{q} \end{cases}$$

$$720 = 2^4 \times 3^2 \times 5 \Rightarrow \frac{p}{q} = \frac{16}{15} \quad b = 720 \times \frac{16}{15} = 768 \Rightarrow \boxed{21}$$

20. Let S be a subset of $\{1, 2, 3, \dots, 2024\}$ such that the following two conditions hold:

- If x and y are distinct elements of S , then $|x - y| > 2$.
- If x and y are distinct odd elements of S , then $|x - y| > 6$.

What is the maximum possible number of elements in S ?

(A) 436 (B) 506 (C) 608 (D) 654 (E) 675

1, 4, 8, 11, 14, 18, 21, 24, 28, ...

$$202 \times 3 + 2 = \boxed{608}$$

21. The numbers, in order, of each row and the numbers, in order, of each column of a 5×5 array of integers form an arithmetic progression of length 5. The numbers in positions (5, 5), (2, 4), (4, 3), and (3, 1) are 0, 48, 16, and 12, respectively. What number is in position (1, 2)?

$$\begin{bmatrix} . & ? & . & . & . \\ . & . & . & 48 & . \\ 12 & . & . & . & . \\ . & . & 16 & . & . \\ . & . & . & . & 0 \end{bmatrix}$$

(A) 19 (B) 24 (C) 29 (D) 34 (E) 39

In row 4, let difference be d , then:

$$(4,4)=16+d, (4,5)=16+2d, (3,5)=32+4d$$

$$\Rightarrow \text{Common difference in Row 3 is } \frac{32+4d-12}{4} = 5+d$$

$$(3,4)=22+2d+(5+d) \\ =27+3d.$$

In column 4,

$$27+3d-48=16+d-(27+3d)$$

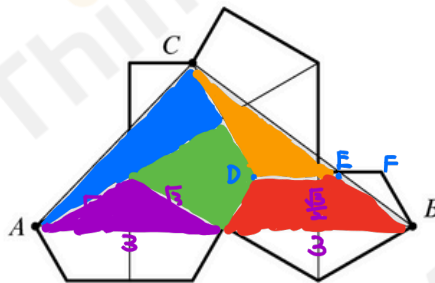
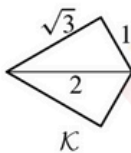
$$5d=10$$

$$d=2$$

Complete table:

$$\begin{bmatrix} 29 & 46 & 63 & 80 \\ & 48 & 60 & \\ 12 & . & . & 33 & 40 \\ & 16 & 18 & 20 \\ & & & & 0 \end{bmatrix}$$

22. Let K be the kite formed by joining two right triangles with legs 1 and $\sqrt{3}$ along a common hypotenuse. Eight copies of K are used to form the polygon shown below. What is the area of triangle ABC ?



(A) $2+3\sqrt{3}$ (B) $\frac{9}{2}\sqrt{3}$ (C) $\frac{10+8\sqrt{3}}{3}$ (D) 8 (E) $5\sqrt{3}$

$$\text{Blue} = \frac{1}{2} \cdot 1 \cdot 2\sqrt{3} = \sqrt{3}, \text{ Purple} = \frac{1}{2} \cdot 3 \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{3}}{4}, \text{ Green} = \sqrt{3}$$

$$\begin{cases} \angle CED \cong \angle BEF \\ \angle CDE \cong \angle F \end{cases} \Rightarrow \triangle CDE \sim \triangle BFE \Rightarrow \text{Red} = \frac{1}{2} \times (3 + \frac{4}{3}) \cdot \frac{\sqrt{3}}{2} = \frac{13}{12}\sqrt{3}$$

$$\Rightarrow \frac{DE}{EF} = \frac{CD}{BF} = \frac{2}{1}$$

$$\therefore DE+EF=DF=2$$

$$\Rightarrow DE = \frac{4}{3}, DF = \frac{2}{3}$$

$$\text{Yellow} = 4 \Delta_{EFB}$$

$$= 4(2\sqrt{3} - \frac{3\sqrt{3}}{4} - \frac{13}{12}\sqrt{3})$$

$$= \frac{2}{3}\sqrt{3}$$

Total:

$$\sqrt{3} + \frac{3\sqrt{3}}{4} + \sqrt{3} + \frac{13}{12}\sqrt{3} + \frac{2}{3}\sqrt{3}$$

$$+ \frac{9}{2}\sqrt{3}$$

23. Integers a , b , and c satisfy $ab + c = 100$, $bc + a = 87$, and $ca + b = 60$. What is $ab + bc + ca$?

- (A) 212 (B) 247 (C) 258 (D) 276 (E) 284

$$(ab+c) - (bc+a) = 100 - 87$$

$$b(a-c) - (a-c) = 13$$

$$(b-1)(a-c) = 1 \times 13$$

$$\Rightarrow b-1 = \pm 2 \text{ or } \pm 13$$

$$\begin{cases} a+c = -17 \\ a-c = -1 \end{cases} \Rightarrow \begin{cases} a = -9 \\ c = -8 \end{cases}$$

$$\Rightarrow \begin{cases} ab = 108 \\ bc = 96 \\ ca = 72 \end{cases}$$

$$ab+bc+ca = 276$$

If $b=2$:

$$(ab+c) + (bc+a) = 100 + 87$$

$$(b+1)(a+c) = 187$$

$$3(a+c) = 187 \Leftarrow \text{Contradict.}$$

If $b=-1$:

$$(b+1)(a+c) = 187$$

$$0(a+c) = 187 \Leftarrow \text{Contradict.}$$

If $b=14$:

$$(b+1)(a+c) = 187$$

$$15(a+c) = 187 \Leftarrow \text{Contradict.}$$

If $b=-12$:

$$(b+1)(a+c) = 187$$

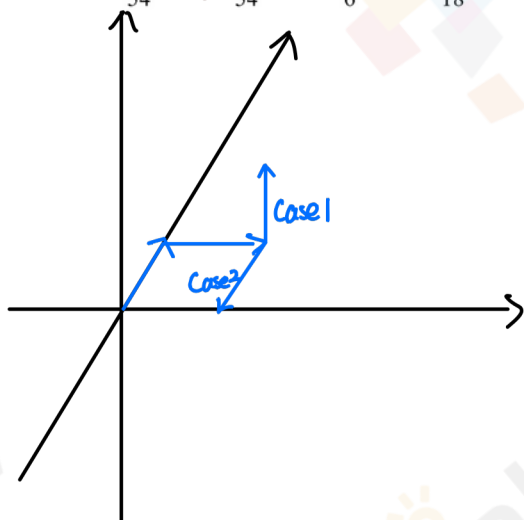
$$-11(a+c) = 187$$

$$a+c = -17$$

$$\text{Works: } \begin{cases} b = -12 \\ a+c = -17 \\ a-c = -1 \end{cases}$$

24. A bee is moving in three-dimensional space. A fair six-sided die with faces labeled A^+ , A^- , B^+ , B^- , C^+ , and C^- is rolled. Suppose the bee occupies the point (a, b, c) . If the die shows A^+ , then the bee moves to the point $(a+1, b, c)$, and if the die shows A^- , then the bee moves to the point $(a-1, b, c)$. Analogous moves are made with the other four outcomes. Suppose the bee starts at the point $(0, 0, 0)$ and the die is rolled four times. What is the probability that the bee traverses four distinct edges of some unit cube?

- (A) $\frac{1}{54}$ (B) $\frac{7}{54}$ (C) $\frac{1}{6}$ (D) $\frac{5}{18}$ (E) $\frac{2}{5}$



First move: any

Second: $\frac{4}{6} = \frac{2}{3}$ (4 cubes share the edge)

Third:

Case 1: Leaves the plane, cube is determined. ($\frac{2}{6} = \frac{1}{3}$)

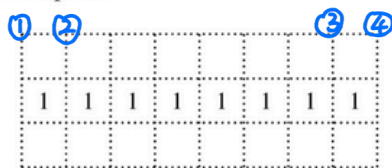
$\Rightarrow$ Fourth: Stay in cube ($\frac{2}{6} = \frac{1}{3}$)

Case 2: Stay in plane ($\frac{1}{6}$)

$\Rightarrow$ Fourth: $\frac{3}{6} = \frac{1}{2}$

$$P = \frac{2}{3} \times \left(\frac{1}{3} \times \frac{1}{3} + \frac{1}{6} \times \frac{1}{2} \right) = \frac{7}{54}$$

25. The figure below shows a dotted grid 8 cells wide and 3 cells tall consisting of $1'' \times 1''$ squares. Carl places 1-inch toothpicks along some of the sides of the squares to create a closed loop that does not intersect itself. The numbers in the cells indicate the number of sides of that square that are to be covered by toothpicks, and any number of toothpicks are allowed if no number is written. In how many ways can Carl place the toothpicks?



First, there are 2 ways;



(A) 130 (B) 144 (C) 146 (D) 162 (E) 196

To create a closed loop in all 3 rows, 1 straight line must be at ① or ②, and 1 straight line at ③ or ④:

Case 1: 2^6



2 ways, for each cell in the middle.

Case 2: 2^5



Case 3: 2^5



Case 4: 2^4



$$\begin{aligned} \text{Total: } & 2^6 + 2^5 + 2^5 + 2^4 + 2 \\ & = 146 \end{aligned}$$



Wechat Account



WhatsApp



Website: <https://www.thethinkacademy.com/>