

2020 AMC 10B

1 What is the value of $1 - (-2) - 3 - (-4) - 5 - (-6)$? (2020 AMC 10B, Question #1)

- A. -20 B. -3 C. 3 D. 5 E. 21

Answer D

Solution We know that when we subtract negative numbers, $a - (-b) = a + b$. The equation becomes $1 + 2 - 3 + 4 - 5 + 6 = (\text{D}) 5$.

2 Carl has 5 cubes each having side length 1, and Kate has 5 cubes each having side length 2. What is the total volume of these 10 cubes? (2020 AMC 10B, Question #2)

- A. 24 B. 25 C. 28 D. 40 E. 45

Answer E

Solution A cube with side length 1 has volume $1^3 = 1$, so 5 of these will have a total volume of $5 \cdot 1 = 5$.

A cube with side length 2 has volume $2^3 = 8$, so 5 of these will have a total volume of $5 \cdot 8 = 40$. $5 + 40 = (\text{E})$

3 The ratio of w to x is $4 : 3$, the ratio of y to z is $3 : 2$, and the ratio of z to x is $1 : 6$. What is the ratio of w to y ? (2020 AMC 10B, Question #3)

- A. $4 : 3$ B. $3 : 2$ C. $8 : 3$ D. $4 : 1$ E. $16 : 3$

Answer E

Solution Solution 1:

WLOG, let $w = 4$ and $x = 3$. Since the ratio of z to x is $1 : 6$, we can substitute in the value of x to get $\frac{z}{3} = \frac{1}{6} \implies z = \frac{1}{2}$. The ratio of y to z is $3 : 2$, so $\frac{y}{\frac{1}{2}} = \frac{3}{2} \implies y = \frac{3}{4}$. The ratio of w to y is then $\frac{4}{\frac{3}{4}} = \frac{16}{3}$ so our answer is (E)

Solution 2:

We need to somehow link all three of the ratios together. We can start by connecting the last two ratios together by multiplying the last ratio by two.

$z : x = 1 : 6 = 2 : 12$, and since $y : z = 3 : 2$, we can link them together to get $y : z : x = 3 : 2 : 12$. Finally, since $x : w = 3 : 4 = 12 : 16$, we can link this again to get: $y : z : x : w = 3 : 2 : 12 : 16$, so $w : y = \text{(E)} 16 : 3$

4 The acute angles of a right triangle are a° and b° , where $a > b$ and both a and b are prime numbers. What is the least possible value of b ? (2020 AMC 10B, Question #4)

- A. 2 B. 3 C. 5 D. 7 E. 11

Answer D

Solution Solution 1:

Since the three angles of a triangle add up to 180° and one of the angles is 90° because it's a right triangle, $a^\circ + b^\circ = 90^\circ$. The greatest prime number less than 90 is 89. If $a = 89^\circ$, then $b = 90^\circ - 89^\circ = 1^\circ$, which is not prime. The next greatest prime number less than 90 is 83. If $a = 83^\circ$, then $b = 7^\circ$, which IS prime, so we have our answer (D) 7

Solution 2:

Looking at the answer choices, only 7 and 11 are coprime to 90. Testing 7, the smaller angle, makes the other angle 83 which is prime, therefore our answer (D) 7

5 How many distinguishable arrangements are there of 1 brown tile, 1 purple tile, 2 green tiles, and 3 yellow tiles in a row from left to right? (Tiles of the same color are indistinguishable.) (2020 AMC 10B, Question #5)

- A. 210 B. 420 C. 630 D. 840 E. 1050

Answer B

Solution Solution 1:

Let's first find how many possibilities there would be if they were all distinguishable, then divide out the ones we overcounted. There are $7!$ ways to order 7 objects. However, since there's $3! = 6$ ways to switch the yellow tiles around without changing anything (since they're indistinguishable) and $2! = 2$ ways to order the green tiles, we have to divide out these possibilities.

Solution 2:

We can repeat chooses extensively to find the answer. There are 7 choose 3 ways to arrange the brown tiles which is 35 . Then from the remaining tiles there are 4 choose $2 = 6$ ways to arrange the red tiles. And now from the remaining two tiles and two slots we can see there are two ways to arrange the purple and brown tiles, giving us an answer of $35 * 6 * 2 = 420$ $\frac{7!}{6 \cdot 2} = (\mathbf{B})420$

6 Driving along a highway, Megan noticed that her odometer showed 15951 (miles). This number is a palindrome-it reads the same forward and backward. Then 2 hours later, the odometer displayed the next higher palindrome. What was her average speed, in miles per hour, during this 2 -hour period?(2020 AMC 10B, Question #6)

- A. 50 B. 55 C. 60 D. 65 E. 70

Answer B

Solution

In order to get the smallest palindrome greater than **15951** , we need to raise the middle digit. If we were to raise any of the digits after the middle, we would be forced to also raise a digit before the middle to keep it a palindrome, making it unnecessarily larger. So we raise **9** to the next largest value, **10** , but obviously, that's not how place value works, so we're in the **16000s** now. To keep this a palindrome, our number is now **16061** . So Megan drove $16061 - 15951 = 110$ miles. Since this happened $\frac{110}{2} = \text{(B)}55$ over 2 hours, she drove at $\bar{2} = \text{(B)} 55$ mph.

7 How many positive even multiples of **3** less than **2020** are perfect squares? (2020 AMC 10B, Question #7)

- A. 7 B. 8 C. 9 D. 10 E. 12

Answer A

Solution Any even multiple of **3** is a multiple of **6** , so we need to find multiples of **6** that are perfect squares and less than **2020** . Any solution that we want will be in the form $(6n)^2$, where n is a positive integer. The smallest possible value is at $n = 1$, and the largest is at $n = 7$ (where the expression equals 1764). Therefore, there are a total of (A) 7 possible numbers.

8 Points P and Q lie in a plane with $PQ = 8$. How many locations for point R in this plane are there such that the triangle with vertices P, Q , and R is a right triangle with area 12 square units?(2020 AMC 10B, Question #8)

- A. 2 B. 4 C. 6 D. 8 E. 12

Answer D

Solution Solution 1:

There are 3 options here: 1. P is the right angle. It's clear that there are 2 points that fit this, one that's directly to the right of P and one that's directly to the left. We don't need to find the length, we just need to know that it is possible, which it is. 2. Q is the right angle. Using the exact same reasoning, there are also 2 solutions for this one. 3. The new point is the right angle. (Diagram temporarily removed due to asymptote error) The diagram looks something like this. We know that the altitude to base $\overline{AB}$ must be 3 since the area is 12. From here, we must see if there are valid triangles that satisfy the necessary requirements. First of all, area. Next, $\overline{BC}^2 + \overline{AC}^2 = 64$ from the Pythagorean Theorem. From here, we must look to see if there are valid solutions. There are multiple ways to do this: Recognizing min and max: We know that the minimum value of $\overline{BC}^2 + \overline{AC}^2 = 64$ is when $\overline{BC} = \overline{AC} = \sqrt{24}$. In this case, the equation becomes $24 + 24 = 48$, which is LESS than 64. $\overline{BC} = 1, \overline{AC} = 24$. The equation becomes $1 + 576 = 577$, which is obviously greater than 64. We can conclude that there are values for $\overline{BC}$ and $\overline{AC}$ in between that satisfy the Pythagorean Theorem. And since $\overline{BC} \neq \overline{AC}$, the triangle is not isosceles, meaning we could reflect it over $\overline{AB}$ and/or the line perpendicular to $\overline{AB}$ for a total of 4 triangles this case.

Solution 2:

Note that line segment $\overline{PQ}$ can either be the shorter leg, longer leg or the hypotenuse. If it is the shorter leg, there are two possible points for Q that can satisfy the requirements - that being above or below $\overline{PQ}$. As such, there are 2 ways for this case. Similarly, one can find that there are also 2 ways for point Q to lie if $\overline{PQ}$ is the longer leg. If it is a hypotenuse, then there are 4 possible points because the arrangement of the shorter and longer legs can be switched, and can be either above or below the line segment. Therefore, the answer is $2 + 2 + 4 = (D)8$

- 9 How many ordered pairs of integers (x, y) satisfy the equation $x^{2020} + y^2 = 2y$? (2020 AMC 10B, Question #9)

- A. 1 B. 2 C. 3 D. 4 E. infinitely many

Answer D

Solution Solution 1: Rearranging the terms and completing the square for y yields the result $x^{2020} + (y - 1)^2 = 1$. Then, notice that x can only be 0, 1 and -1 because any value of x^{2020} that is greater than 1 will cause the term $(y - 1)^2$ to be less than 0, which is impossible as Y must be real. Therefore, plugging in the above values for X gives the ordered pairs (0, 0), (1, 1), $(-1, 1)$, and (0, 2) gives a total of (D) 4 ordered pairs.

Solution 2:

Bringing all of the terms to the LHS, we see a quadratic equation $y^2 - 2y + x^{2020} = 0$ in terms of y . Applying the quadratic formula, we

get $y = \frac{2 \pm \sqrt{4 - 4 \cdot 1 \cdot x^{2020}}}{2} = \frac{2 \pm \sqrt{4(1 - x^{2020})}}{2}$ in order for y to be real, which it must be given the stipulation that we are seeking integral answers, we know that the discriminant, $4(1 - x^{2020})$ must be nonnegative. Therefore,

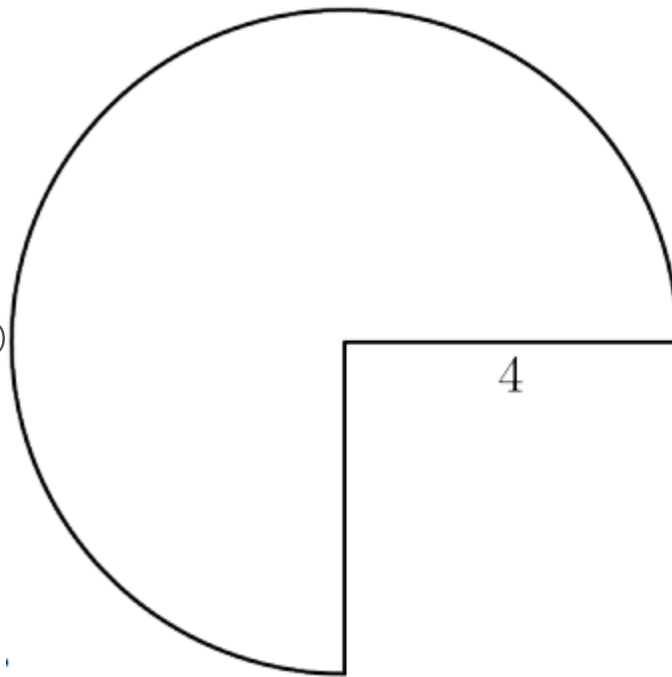
$4(1 - x^{2020}) \geq 0 \implies x^{2020} \leq 1$ Here, we see that we must split the inequality into a compound, resulting in $-1 \leq x \leq 1$ The only integers that satisfy this are $x \in \{-1, 0, 1\}$. Plugging these values back into the quadratic equation, we see that $x = \{-1, 1\}$

both produce a discriminant of 0, meaning that there is only 1 solution for Y . If $x = \{0\}$, then the discriminant is nonzero, therefore resulting in two solutions for Y .

Thus, the answer is $2 \cdot 1 + 1 \cdot 2 = \text{(D)} 4$

- 10 A three-quarter sector of a circle of radius 4 inches together with its interior can be rolled up to form the lateral surface area of a right circular cone by taping together along the two radii shown. What is the volume of the cone in cubic inches? (2020 AMC

10B, Question #10)



A. $3\pi\sqrt{5}$

B. $4\pi\sqrt{3}$

C. $3\pi\sqrt{7}$

D. $6\pi\sqrt{3}$

E. $6\pi\sqrt{7}$

Answer C

Solution Solution 1 : Notice that when the cone is created, the radius of the circle will become the slant height of the cone and the intact circumference of the circle will become the circumference of the base of the cone. We can calculate that the intact circumference of the circle is $8\pi \cdot \frac{3}{4} = 6\pi$. Since that is also equal to the circumference of the cone, the radius of the cone is 3 . We also have that the slant height of the cone is 4 . Therefore, we use the Pythagorean Theorem to calculate that the height of the cone is $\sqrt{4^2 - 3^2} = \sqrt{7}$. The volume of the cone is $\frac{1}{3} \cdot \pi \cdot 3^2 \cdot \sqrt{7} = \text{(C)} 3\pi\sqrt{7}$

Solution 2 (Last Resort/Cheap):

Using a ruler, measure a circle of radius 4 and cut out the circle and then the quarter missing. Then, fold it into a cone and measure the diameter to be 6 cm $\Rightarrow r = 3$. You can form a right triangle with sides 3,4 , and then through the Pythagorean theorem the height h is found to be $h^2 = 4^2 - 3^2 \Rightarrow h = \sqrt{7}$. The volume of a cone is $\frac{1}{3}\pi r^2 h$. Plugging in we find $V = 3\pi\sqrt{7} \Rightarrow \text{(C)}$

- 11 Ms. Carr asks her students to read any 5 of the 10 books on a reading list. Harold randomly selects 5 books from this list, and Betty does the same. What is the probability that there are exactly 2 books that they both select?(2020 AMC 10B, Question #11)

A. $\frac{1}{8}$ B. $\frac{5}{36}$ C. $\frac{14}{45}$ D. $\frac{25}{63}$
 E. $\frac{1}{2}$

Answer D

Solution We don't care about which books Harold selects. We just care that Betty picks 2 books from Harold's list and 3 that aren't on Harold's list. The total amount of combinations of books that Betty can select $\binom{10}{5} = 252$ $\binom{5}{2} = 10$ ways for Betty to choose 2 of the books that are on Harold's list. From the remaining 5 books that aren't on Harold's list, there $\binom{5}{3} = 10$ $\frac{10 \cdot 10}{252} = \text{(D)} \frac{25}{63}$

- 12 The decimal representation of $\frac{1}{20^{20}}$ consists of a string of zeros after the decimal point, followed by a 9 and then several more digits. How many zeros are in that initial string of zeros after the decimal point?(2020 AMC 10B, Question #12)

A. 23 B. 24 C. 25 D. 26 E. 27

Answer D

Solution Solution 1:

$\frac{1}{20^{20}} = \frac{1}{(10 \cdot 2)^{20}} = \frac{1}{10^{20} \cdot 2^{20}}$ Now we do some estimation. Notice that $2^{20} = 1024^2$, which means that 2^{20} is a little more than $1000^2 = 1,000,000$. Multiplying it with 10^{20} , we get that the denominator is about $\underbrace{100 \dots 0}_{26 \text{ zeros}}$. Notice that when we divide 1 by an n digit number, there are $n - 1$ zeros before the first nonzero digit. This means that when we divide 1 by the 27 digit $\underbrace{100 \dots 0}_{26 \text{ zeros}}$, there are integer decimal point.

Solution 2:

First rewrite $\frac{1}{20^{20}}$ as $\frac{5^{20}}{10^{40}}$. Then, we know that when we write this in decimal form, there will be 40 digits after the decimal point. Therefore, we just have to find the number of digits in 5^{20} . $\log 5^{20} = 20 \log 5$ and knowing $\log 5 \approx 0.69$ (alternatively use the fact that $\log 5 = 1 - \log 2$), $\lfloor 20 \log 5 \rfloor + 1 = \lfloor 20 \cdot 0.69 \rfloor + 1 = 13 + 1 = 14$ digits. Our answer is (D) 26

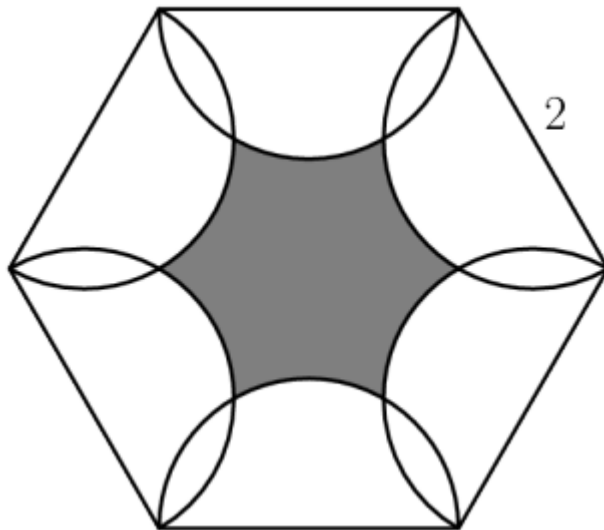
13 Andy the Ant lives on a coordinate plane and is currently at $(-20, 20)$ facing east (that is, in the positive x -direction). Andy moves 1 unit and then turns 90 degrees left. From there, Andy moves 2 units (north) and then turns 90° degrees left. He then moves 3 units (west) and again turns 90° degrees left. Andy continues his progress, increasing his distance each time by 1 unit and always turning left. What is the location of the point at which Andy makes the ~~2020th~~ left turn?(2020 AMC 10B, Question #13)

- A. $(-1030, -994)$ B. $(-1020, -994)$ C. $(-1126, -994)$ D. $(-1024, -9991)$
E. $(-1022, -994)$

Answer B

Solution You can find that every four moves both coordinates decrease by 2. Therefore, both coordinates need to decrease by two ~~505~~ times. You subtract, giving you the answer of (B) $(-1030, -990)$

14 As shown in the figure below, six semicircles lie in the interior of a regular hexagon with side length 2 so that the diameters of the semicircles coincide with the sides of the hexagon. What is the area of the shaded region - inside the hexagon but outside all of the semicircles?(2020 AMC 10B, Question #14)



A. $6\sqrt{3} - 3\pi$

B. $\frac{9\sqrt{3}}{2} - 2\pi$

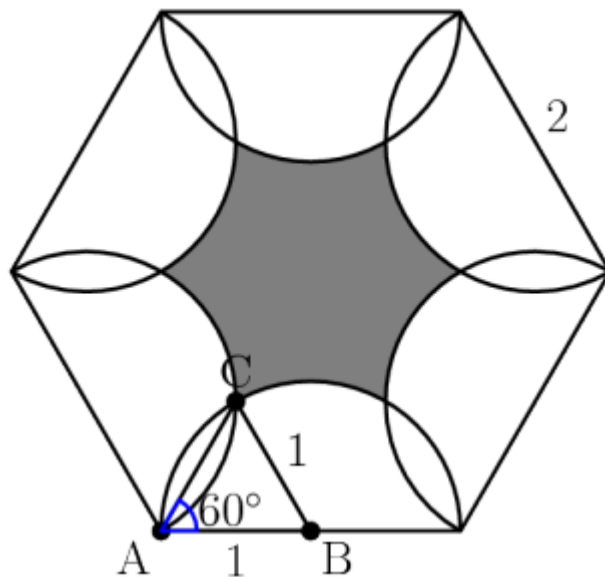
C. $\frac{3\sqrt{3}}{2} - \frac{\pi}{3}$

D. $3\sqrt{3} - \pi$

E. $\frac{9\sqrt{3}}{2} - \pi$

Answer D

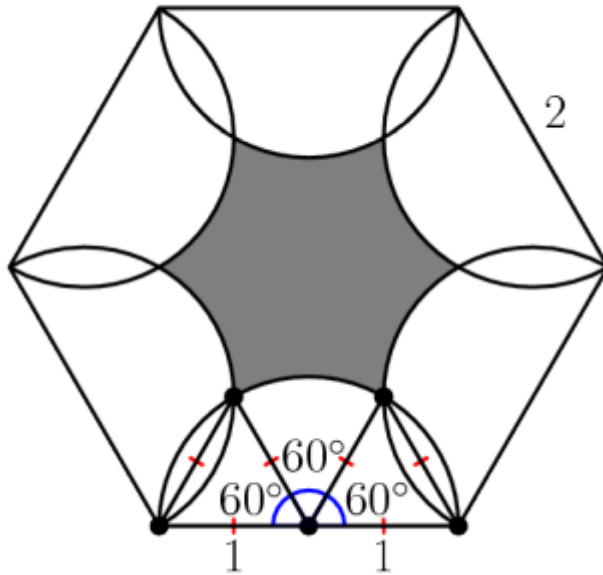
Solution Solution 1:



Let point A be a vertex of the

regular hexagon, let point B be the midpoint of the line connecting point A and a neighboring vertex, and let point C be the second intersection of the two semicircles that pass through point A . Then, $BC = 1$, since B is the center of the semicircle with radius 1 that C lies on, $AB = 1$, since B is the center of the semicircle with radius 1 that A lies on, and $\angle BAC = 60^\circ$, as a regular hexagon has angles of 120° , and $\angle BAC$

is half of any angle in this hexagon. Now, using the sine law, $\frac{1}{\sin \angle ACB} = \frac{1}{\sin 60^\circ}$, so $\angle ACB = 60^\circ$. Since the angles in a triangle sum to 180° , $\angle ABC$ is also 60° . Therefore, $\triangle ABC$ is an equilateral triangle with side lengths of 1.

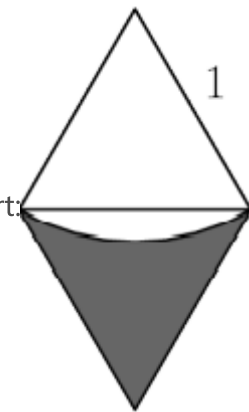
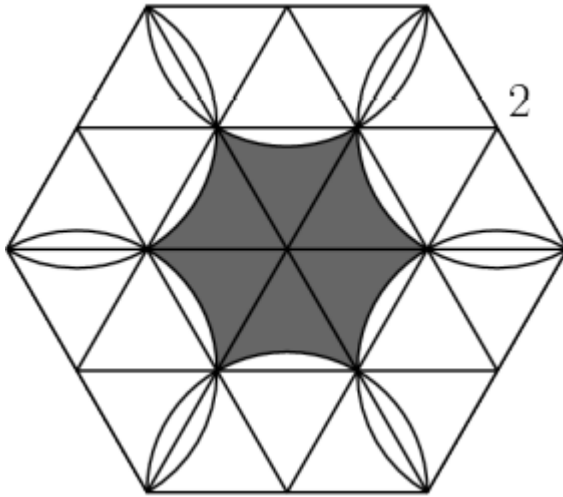


Since the area of a regular hexagon

can be found with the formula $\frac{3\sqrt{3}s^2}{2}$, where s is the side length of the hexagon, the area of this hexagon $\frac{3\sqrt{3}(2^2)}{2} = 6\sqrt{3}$. Since the area of an equilateral triangle can be found with the formula $\frac{\sqrt{3}}{4}s^2$, where s is the side length of the equilateral triangle, the area of an equilateral triangle with side lengths of 1 is $\frac{\sqrt{3}}{4}(1^2) = \frac{\sqrt{3}}{4}$. Since the area of a circle can be found with the formula πr^2 , the area of a sixth of a circle with radius 1 is $\frac{\pi(1^2)}{6} = \frac{\pi}{6}$. In each sixth of the hexagon, there of a circle with radius 1 is $6 \cdot \frac{\pi}{6}$. In each sixth of the hexagon, there sixth of a circle with radius 1 colored white, with an area of $\frac{\pi}{6}$. The rest of the sixth is colored gray. Therefore, the total area that is colored white in each sixth of the hexagon is $2 \left(\frac{\sqrt{3}}{4} \right) + \frac{\pi}{6}$, which equals $\frac{\sqrt{3}}{2} + \frac{\pi}{6}$, and the total area colored white is $6 \left(\frac{\sqrt{3}}{2} + \frac{\pi}{6} \right)$, which equals $3\sqrt{3} + \pi$. Since the area colored gray equals the total area of the hexagon minus the area colored white, the area colored gray is $6\sqrt{3} - (3\sqrt{3} + \pi)$, which equals (D) $3\sqrt{3} - \pi$

Solution 2:

First, subdivide the hexagon into 24 equilateral triangles with side length 1 :



Now note that the entire shaded region is just 6 times this part:

The entire rhombus is just 2 equilateral triangles with side lengths of 1, so it has an area of: $2 \cdot \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$ area of: $\frac{1}{6} \cdot \pi \cdot 1^2 = \frac{\pi}{6}$ Hence, the area of the shaded region in that section is $\frac{\sqrt{3}}{2} - \frac{\pi}{6}$ For a final area $6 \left(\frac{\sqrt{3}}{2} - \frac{\pi}{6} \right) = 3\sqrt{3} - \pi \Rightarrow \text{(D)}$

- 15 Steve wrote the digits 1, 2, 3, 4, and 5 in order repeatedly from left to right, forming a list of 10,000 digits, beginning 123451234512... He then erased every third digit from his list (that is, the 3rd, 6th, 9th, ... digits from the left), then erased every fourth digit from the resulting list (that is, the 4th, 8th, 12th, ..., digits from the left in what remained), and then erased every fifth digit from what remained at that point. What is the sum of the three digits that were then in the positions 2019, 2020, 2021? (2020 AMC 10B, Question #15)

A. 7 B. 9 C. 10 D. 11 E. 12

Answer D

Solution After erasing every third digit, the list becomes **1245235134** . . . repeated. After erasing every fourth digit from this list, the list becomes **124235341452513** . . repeated. Finally, after erasing every fifth digit from this list, the list becomes **24253415251** . . repeated. Since this list repeats every 12 digits and since 2019, 2020, 2020 are 3, 4, 5 respectively in (mod 12), we have that the 2019th, 2020th, and 2021st digits are the 3rd, 4th, and 5th digits respectively. It follows that the answer is $4 + 2 + 5 = |(D) \perp 1|$.

16 Bela and Jenn play the following game on the closed interval $[0, n]$ of the real number line, where n is a fixed integer greater than 4 . They take turns playing, with Bela going first. At his first turn, Bela chooses any real number in the interval $[0, n]$. Thereafter, the player whose turn it is chooses a real number that is more than one unit away from all numbers previously chosen by either player. A player unable to choose such a number loses. Using optimal strategy, which player will win the game?(2020 AMC 10B, Question #16)

- A. Bela will always win.
- B. Jenn will always win
- C. Bela will win if and only if n is odd
- D. Jenn will win if and only if n is odd.
- E. Jenn will win if and only if $n > 8$.

Answer A

Solution Solution 1:

Notice that to use the optimal strategy to win the game, Bela must select the middle number in the range $[0, n]$ and then mirror whatever number Jenn selects. Therefore, if Jenn can select a number within the range, so can Bela. Jenn will always be the first person to run out of a number to choose, so the answer is (A)Bela will always win.

Solution 2 (Guessing): First of all, realize that the value of n should have no effect on the strategy at all. This is because they can choose real numbers, not integers, so even if n is odd, for example, they can still go halfway. Similarly, there is no reason

the strategy would change when $n > 8$.

So we are left with (A) and (B). From here it is best to try out random numbers and try to find the strategy that will let Bela win, but if you can't find it, realize that it is more likely the answer is (A) Bela will always win since Bela has the first move and thus has more control.

17 There are 10 people standing equally spaced around a circle. Each person knows exactly 3 of the other people: the 2 people standing next to her or him as well as the person directly across the circle. How many ways are there for the 10 people to split up into 5 pairs so that the members of each pair know each other?(2020 AMC 10B, Question #17)

- A. 11 B. 12 C. 13 D. 14 E. 15

Answer C

Solution Let us use casework on the number of diagonals. Case 1 : 0 diagonals There are 2 ways: either 1 pairs with 2, 3 pairs with 4, and so on or 10 pairs with 1, 2 pairs with 3, etc.

Case 2 : 1 diagonal There are 5 possible diagonals to draw (everyone else pairs with the person next to them. Note that there cannot be 2 diagonals. Case 3 : 3 diagonals Note that there cannot be a case with 4 diagonals because then there would have to be 5 diagonals for the two remaining people, thus a contradiction. Case 4 : 5 diagonals There is 1 way to do this. Thus, in total there are $2 + 5 + 5 + 1 = 13$ possible ways.

18 An urn contains one red ball and one blue ball. A box of extra red and blue balls lie nearby. George performs the following operation four times: he draws a ball from the urn at random and then takes a ball of the same color from the box and returns those two matching balls to the urn. After the four iterations the urn contains six balls. What is the probability that the urn contains three balls of each color?(2020 AMC 10B, Question

#18)

A. $\frac{1}{6}$

B. $\frac{1}{5}$

C. $\frac{1}{4}$

D. $\frac{1}{3}$

E. $\frac{1}{2}$

Answer B

Solution Solution 1:

Let R denote that George selects a red ball and B that he selects a blue one. Now, in order to get 3 balls of each color, he needs 2 more of both R and B . There are 6 cases: $RRBB, RBRB, RBBR, BBRR, BRBR, BRRB$ (we can confirm that there are only 6 since $\binom{4}{2} = 6$). However we can clump $RRBB + BBRR, RBRB + BRBR$ and $RBBR + BRRB$ together since they are equivalent by symmetry.

CASE 1: $RRBB$ and $BBRR$ Let's find the probability that he picks the balls in the order of $RRBB$. The probability that the first ball he picks is red is $\frac{1}{2}$. Now there are 2 reds and 1 blue in the urn. The probability that he picks red again is now $\frac{2}{3}$. There are 3 reds and 1 blue now. The probability that he picks a blue is $\frac{1}{4}$. Finally, there are 3 reds and 2 blues. The probability that he picks a blue is $\frac{1}{5}$. So the probability that the $RRBB$ case happens is $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{4} \cdot \frac{1}{5} = \frac{1}{60}$. However, since the $BBRR$ case is the exact same by symmetry, case 1 has a probability of $\frac{1}{60} \cdot 2 = \frac{1}{30}$ chance of happening.

CASE 2: $RBRB$ and $BRBR$ Let's find the probability that he picks the balls in the order of $RBRB$. The probability that the first ball he picks is red is $\frac{1}{2}$. Now there are 2 reds and 1 blue in the urn. The probability that he picks blue is $\frac{1}{3}$. There are 2 reds and 2 blues now. The probability that he picks a red is $\frac{1}{2}$. Finally, there are 3 reds and 2 blues. The probability that he picks a blue is $\frac{2}{5}$. So the probability that the $RBRB$ case happens $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{2}{5} = \frac{1}{60}$. However, since the $BRBR$ case is the exact same by symmetry, case 2 has a probability of $\frac{1}{60} \cdot 2 = \frac{1}{30}$ chance of happening.

CASE 3: $RBBR$ and $BRRB$ Let's find the probability that he picks the balls in the order of $RBBR$. The probability that the first ball he picks is red is $\frac{1}{2}$. Now there are 2 reds and 1 blue in the urn. The probability that he picks blue is $\frac{1}{3}$.

There are 2 reds and 2 blues now. The probability that he picks a blue is $\frac{1}{2}$. Finally, there are 2 reds and 3 blues. The probability that he picks a red is $\frac{2}{5}$. So the probability that the **RBBR** case happens is $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{2}{5} = \frac{1}{30}$. However, since the **BRBR** case is the exact is $\frac{2}{3} \cdot \frac{2}{5} \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{30}$. However, since the **BRBR** case is the exact same by symmetry, case 3 has a probability of $\frac{1}{30} \cdot 2 = \frac{1}{15}$ chance of happening.

Adding up the cases, we have $\frac{1}{15} + \frac{1}{15} + \frac{1}{15} = \text{(B)} \frac{1}{5}$

Solution 2: We know that we need to find the probability of adding 2 red and 2 blue balls in some order. There are 6 ways to do this, since there are $\binom{4}{2} = 6$ ways to arrange **RRBB** in some order. We will show that the probability for each of these 6 ways is the same.

We first note that the denominators should be counted by the same number. This number is $2 \cdot 3 \cdot 4 \cdot 5 = 120$. This is because 2, 3, 4, and 5 represent how many choices there are for the four steps. No matter what the $k - \text{th}$ step involves $k + 1$ numbers to choose from. The numerators are the number of successful operations. No matter the order, the first time a red is added will come from 1 choice and the second time will come from 2 choices, since that is how many reds are in the urn originally. The same goes for the blue ones. The numerator must equal $(1 \cdot 2)^2$. Therefore, the probability for each of the orderings of **RRBB** is $\text{(B)} \frac{1}{5} \cdot \frac{4}{120} = \frac{1}{30}$. There are 6 of these, so the total probability is $\text{(B)} \frac{1}{5}$

19 In a certain card game, a player is dealt a hand of 10 cards from a deck of 52 distinct cards. The number of distinct (unordered) hands that can be dealt to the player can be written as $158A00\sim A4AA0$. What is the digit A ? (2020 AMC 10B, Question #19)

- A. 2 B. 3 C. 4 D. 6 E. 7

Answer A

Solution Solution 1:

$158A00A4A40 \equiv 1 + 5 + 8 + A + 0 + 0 + A + 4 + A + A + 0 \equiv 4A \pmod{9}$ We're looking for the amount of ways we can get 10 cards from a deck of 52, which is represented by $\binom{52}{10} \cdot \binom{52}{10} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44 \cdot 43}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$ We need to get rid of the multiples of 3, which will subsequently get rid of the multiples of 9 (if we didn't, the zeroes would mess with the equation since you can't divide by 0)

$$9 \cdot 5 = 45, 8 \cdot 6 = 48, \frac{51}{3} \text{ leaves us with } 17. \frac{52 \cdot 51^{17} \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44 \cdot 43}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

Converting these into $\pmod{9}$, we have

$$\binom{52}{10} \equiv \frac{(-2) \cdot (-1) \cdot (-4) \cdot 4 \cdot 2 \cdot 1 \cdot (-1) \cdot (-2)}{1 \cdot (-2) \cdot 4 \cdot 2 \cdot 1} \equiv (-1) \cdot (-4) \cdot (-1) \cdot (-2) \equiv 8 \pmod{9};$$

$$4A \equiv 8 \pmod{9} \implies A = (A)2$$

Solution 2:

$\binom{52}{10} = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44 \cdot 43}{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 26 \cdot 17 \cdot 5 \cdot 7 \cdot 47 \cdot 46 \cdot 11 \cdot 43$ Since this number is divisible by 4 but not 8, the last 2 digits must be divisible by 4 but the last 3 digits cannot be divisible by 8. This narrows the options down to 2 and 6. Also, the number cannot be divisible by 3. Adding up the digits, we get $18 + 4A$. If $A = 6$, then the expression equals 42, a multiple of 3. This would mean that the entire number would be divisible by 3, which is not what we want. Therefore, the only option is (A)2.

- 20 Let B be a right rectangular prism (box) with edges lengths 1, 3, and 4, together with its interior. For real $r \geq 0$, let $S(r)$ be the set of points in 3 dimensional space that lie within a distance r of some point B . The volume of $S(r)$ can be expressed as $ar^3 + br^2 + cr + d$, where a, b, c , and d are positive real numbers. What is $\frac{bc}{ad}$? (2020 AMC 10B, Question #20)

A. 6 B. 19 C. 24 D. 26 E. 38

Answer B

Solution

Split $S(r)$ into 4 regions: 1. The rectangular prism itself 2. The extensions of the faces of B 3. The quarter cylinders at each edge of B 4. The one-eighth spheres at each corner of B

Region 1: The volume of B is 12, so $d = 12$ Region 2: The volume is equal to the surface area of B times r . The surface area can be computed to be

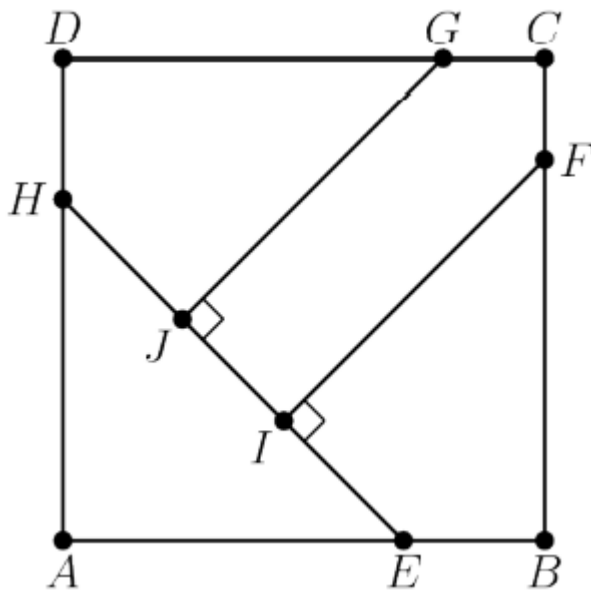
$2(4 * 3 + 3 * 1 + 4 * 1) = 38$, so $c = 38$. Region 3 : The volume of each quarter cylinder is equal to $(\pi * r^2 * h) / 4$. The sum of all such cylinders must equal $(\pi * r^2) / 4$ times the sum of the edge lengths. This can be computed as $4(4 + 3 + 1) = 32$, so the sum of the volumes of the quarter cylinders is $8\pi * r^2$, so $b = 8\pi$

Region 4: There is an eighth of a sphere of radius r at each corner. Since there are 8 corners, these add up to one full sphere of radius r . The volume of this sphere is

$\frac{4}{3}\pi * r^3$, so $a = \frac{4\pi}{3}$. Using these values, $\frac{(8\pi)(38)}{(4\pi/3)(12)} = (\mathbf{B})19$

21 In square $ABCD$, points E and H lie on $\overline{AB}$ and $\overline{DA}$, respectively, so that $AE = AH$.

Points F and G lie on $\overline{BC}$ and $\overline{CD}$, respectively, and points I and J lie on $\overline{EH}$ so that $\overline{FI} \perp \overline{EH}$ and $\overline{GJ} \perp \overline{EH}$. See the figure below. Triangle AEH , quadrilateral $BFIE$, quadrilateral $DHJG$, and pentagon $FCGJI$ each has area 1. What is FI^2 ? (2020 AMC 10B, Question #21)



A. $\frac{7}{3}$

C. $1 + \sqrt{2}$

E. $2\sqrt{2}$

B. $8 - 4\sqrt{2}$

D. $\frac{7}{4}\sqrt{2}$

Answer B

Solution Solution 1:

Since the total area is 4, the side length of square $ABCD$ is 2. We see that since triangle HAE is a right isosceles triangle with area 1, we can determine sides HA and AE both to be $\sqrt{2}$. Now, consider extending FB and IE until they intersect. Let the point of intersection be K . We note that EBK is also a right isosceles triangle with side $2 - \sqrt{2}$ and find its area to be $3 - 2\sqrt{2}$. Now, we notice that FIK is also a right isosceles triangle and find its area to be $\frac{1}{2}FI^2$. This is also equal to $1 + 3 - 2\sqrt{2}$ or $4 - 2\sqrt{2}$. Since we are looking for FI^2 , we want two times this. That gives (B) $8 - 4\sqrt{2}$

Solution 2:

Since this is a geometry problem involving sides, and we know that HE is 2, we can use our ruler and find the ratio between FI and HE . Measuring (on the booklet), we get that HE is about 1.8 inches and FI is about 1.4 inches. Thus, we can then multiply the length of HE by the ratio of $\frac{1.4}{1.8}$, of which we then get $FI = \frac{14}{9}$. We take the square of that and of $\frac{196}{81}$, and the closest answer to that is (B) $8 - 4\sqrt{2}$. (Note that this is just a strategy I happened to use that worked. Do not press your luck with this strategy, for it was a lucky guess)

22 What is the remainder when $2^{202} + 202$ is divided by $2^{101} + 2^{51} + 1$? (2020 AMC 10B, Question #22)

- A. 100 B. 101 C. 200 D. 201 E. 202

Answer D

Solution Solution 1: Let $x = 2^{50}$. We are now looking for the remainder of $\frac{4x^4 + 202}{2x^2 + 2x + 1}$. We could proceed with polynomial division, but the denominator looks awfully similar to

the Sophie Germain Identity, which states that

$a^4 + 4b^4 = (a^2 + 2b^2 + 2ab)(a^2 + 2b^2 - 2ab)$ Let's use the identity, with $a = 1$ and $b = x$, so we have $1 + 4x^4 = (1 + 2x^2 + 2x)(1 + 2x^2 - 2x)$ Rearranging, we can see that this is

exactly what we need: $\frac{4x^4 + 1}{2x^2 + 2x + 1} = 2x^2 - 2x + 1$ Since the first
 So $\frac{4x^4 + 202}{2x^2 + 2x + 1} = \frac{4x^4 + 1}{2x^2 + 2x + 1} + \frac{201}{2x^2 + 2x + 1}$
 half divides cleanly as shown earlier, the remainder must be (D) 201.

Solution 2:

Similar to Solution 1, let $x = 2^{50}$. It suffices to find remainder of $\frac{4x^4 + 202}{2x^2 + 2x + 1}$ of $2x^2 + 2x + 1$. Dividing polynomials results in a remainder of (D) 201

23 Square $ABCD$ in the coordinate plane has vertices at the points

$A(1, 1)$, $B(-1, 1)$, $C(-1, -1)$, and $D(1, -1)$. Consider the following four transformations: L , a rotation of 90° counterclockwise around the origin; R , a rotation of 90° clockwise around the origin; H , a reflection across the X -axis; and V , a reflection across the Y -axis. Each of these transformations maps the squares onto itself, but the positions of the labeled vertices will change. For example, applying R and then V would send the vertex A at $(1, 1)$ to $(-1, -1)$ and would send the vertex B at $(-1, 1)$ to itself. How many sequences of 20 transformations chosen from $\{L, R, H, V\}$ will send all of the labeled vertices back to their original positions? (For example, R, R, V, H is one sequence of 4 transformations that will send the vertices back to their original positions.)(2020 AMC 10B, Question #23)

- A. 2^{37} B. $3 \cdot 2^{36}$ C. 2^{38} D. $3 \cdot 2^{37}$
 E. 2^{39}

Answer C

Solution Solution 1: Let (+) denote counterclockwise/starting orientation and (-) denote clockwise orientation. Let 1, 2, 3, and 4 denote which quadrant A is in.

Realize that from any odd quadrant and any orientation, the 4 transformations result in some permutation of $(2+, 2-, 4+, 4-)$. The same goes that from any even quadrant and any orientation, the 4 transformations result in some permutation of $(1+, 1-, 3+, 3-)$. We start our first 19 moves by doing whatever we want, 4 choices each time. Since 19 is odd, we must end up on an even quadrant.

As said above, we know that exactly one of the four transformations will give us $(1+)$, and we must use that transformation. Thus $4^{19} = (C)2^{38}$

Solution 2: Hopefully, someone will think of a better one, but here is an indirect answer, use only if you are really desperate. 20 moves can be made, and each move have 4 choices, so a total of $4^{20} = 2^{40}$ moves. First, after the 20 moves, Point A can only be in first quadrant $(1, 1)$ or third quadrant $(-1, -1)$. Only the one in the first quadrant works, so divide by 2. Now, C must be in the opposite quadrant as A . B can be either in the second $(-1, 1)$ or fourth quadrant $(1, -1)$, but we want it to be in the second quadrant, so divide by 2 again. Now as A and B satisfy the conditions, C and D will also be at their original spot. $\frac{2^{40}}{2 \cdot 2} = 2^{38}$. The answer is C

24 How many positive integers n satisfy $\frac{n + 1000}{70} = \lfloor \sqrt{n} \rfloor$? (Recall that $\lfloor x \rfloor$ is the greatest integer less than or equal to x .) (2020 AMC 10B, Question #24)

- A. 2 B. 4 C. 6 D. 30 E. 32

Answer C

Solution Solution 1 :

First notice that the graphs of $(x + 1000)/70$ and $\sqrt{n}$ intersect at 2 points. Then, notice that $(n + 1000)/70$ must be an integer. This means that n is congruent to $50 \pmod{70}$. For the first intersection, testing the first few values of n (adding 70 to n each time and noticing the left side increases by 1 each time) yields $n = 20$ and $n = 21$. Starting from the graph can narrow down the other cases, being $n = 47, 50$. This results

in a total of 6 cases, for an answer of **(C)6**

Solution 2 (Graphing):

One intuitive approach to the question is graphing. Obviously, you should know what the graph of the square root function is, and if any function is floored (meaning it is taken to the greatest integer less than a value), a stair-like figure should appear. The other function is simply a line with a slope of $1/70$. If you precisely draw out the two regions of the graph where the derivative of the square function nears the derivative of the linear function, you can now deduce that 3 values of intersection lay closer to the left side of the stair, and 3 values lay closer to the right side of the stair. With meticulous graphing, you can realize that the answer is **(C)6**. A in-depth graph with intersection points is linked below.

<https://www.desmos.com/calculator/e5wk9adbuk>

25 Let $D(n)$ denote the number of ways of writing the positive integer n as a product $n = f_1 \cdot f_2 \cdots f_k$, where $k \geq 1$, the f are integers strictly greater than 1, and the order in which the factors are listed matters (that is, two representations that differ only in the order of the factors are counted as distinct). For example, the number 6 can be written as $6, 2 \cdot 3$, and $3 \cdot 2$, so $D(6) = 3$. What is $D(96)$? (2020 AMC 10B, Question #25)

- A. 112 B. 128 C. 144 D. 172 E. 184

Answer A

Solution Solution 1:

Note that $96 = 2^5 \cdot 3$. Since there are at most six not necessarily distinct factors > 1 multiplying to 96, we have six cases. $k = 1, 2, \dots, 6$. Now we look at each of the six cases. $k = 1$: we see that there is 1 way, merely 96. $k = 2$: This way, we have the 3 in one slot and 2 in another, and symmetry. The four other 2's leave us with 5 ways and symmetry doubles us so we have 10. $k = 3$: We have $3, 2, 2$ as our baseline. We

need to multiply by 2 in 3 places, and see that we can split the remaining three powers of 2 in a manner that is $3 - 0 - 0$, $2 - 1 - 0$ or $1 - 1 - 1$. A $3 - 0 - 0$ split has $6 + 3 = 9$ ways of happening ($24 - 2 - 2$ and symmetry; $2 - 3 - 16$ and symmetry), a $2 - 1 - 0$ split has $6 \cdot 3 = 18$ ways of happening (due to all being distinct) and a $1 - 1 - 1$ split has 3 ways of happening ($6 - 4 - 4$ and symmetry) so in this case we have $9 + 18 + 3 = 30$ ways. $k = 4$: We have $3, 2, 2, 2$ as our baseline, and for the two other 2s, we have a $2 - 0 - 0 - 0$ or $1 - 1 - 0 - 0$ split. The former grants us $4 + 12 = 16$ ways ($12 - 2 - 2 - 2$ and symmetry and $3 - 8 - 2 - 2$ and symmetry) and the latter grants us also $12 + 12 = 24$ ways ($6 - 4 - 2 - 2$ and symmetry and $3 - 4 - 4 - 2$ and symmetry) for a total of $16 + 24 = 40$ ways. $k = 5$: We have $3, 2, 2, 2, 2$ as our baseline and one place to put the last two: on another two or on the three. On the three gives us 5 ways due to symmetry and on another two gives us $5 \cdot 4 = 20$ ways due to symmetry. Thus, we have $5 + 20 = 25$ ways. $k = 6$: We have $3, 2, 2, 2, 2, 2$ and symmetry and no more twos to multiply, so by symmetry, we have 6 ways. Thus, adding, we have $1 + 10 + 30 + 40 + 25 + 6 = \text{(A)} 112$

Solution 2:

As before, note that $96 = 2^5 \cdot 3$, and we need to consider 6 different cases, one for each possible value of k , the number of factors in our factorization. However, instead of looking at each individually, find a general form for the number of possible factorizations with k factors. First, the factorization needs to contain one factor that is itself a multiple of 3, and there are k to choose from, and the rest must contain at least one factor of 2. Next, consider the remaining $6 - k$ factors of 2 left to assign to the k factors. Using stars and bars, the number of ways to do this

$\binom{(6-k)+k-1}{6-k} = \binom{5}{6-k}$ possibilities for each k . To obtain the total number of factorizations, add all possible values for k :

$$\sum_{k=1}^6 k \binom{5}{6-k} = 1 + 10 + 30 + 40 + 25 + 6 = \text{(A)} 112$$