

2017 AMC 8

1 Which of the following values is largest?

A. $2 + 0 + 1 + 7$

B. $2 \times 0 + 1 + 7$

C. $2 + 0 \times 1 + 7$

D. $2 + 0 + 1 \times 7$

E. $2 \times 0 \times 1 \times 7$

Answer A

Solution **Solution 1**

We will compute each expression.

A) $2 + 0 + 1 + 7 = 10$

B) $2 \times 0 + 1 + 7 = 8$

C) $2 + 0 \times 1 + 7 = 9$

D) $2 + 0 + 1 \times 7 = 9$

E) $2 \times 0 \times 1 \times 7 = 0$

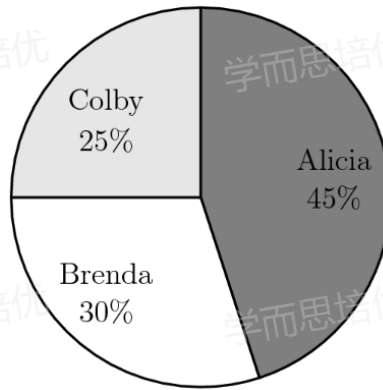
Ordering these, we get 10, 8, 9, 9, 0. Out of these, 10 is the largest number and option (A)

adds up to 10. Therefore, the answer is **(A) $2 + 0 + 1 + 7$** .

Solution 2

We immediately see that every one of the choices, except for A and D, has a number multiplied by 0. This will only make the expression's value smaller. We are left with A and D, but in D, 1 is multiplied by 7 to get 7, whereas in answer choice A, we get 8 out of 7 and 1, instead of 7. Therefore, **(A) $2 + 0 + 1 + 7$** is your answer.

2 Alicia, Brenda, and Colby were the candidates in a recent election for student president. The pie chart below shows how the votes were distributed among the three candidates. If Brenda received 36 votes, then how many votes were cast all together?



- A. 70 B. 84 C. 100 D. 106 E. 120

Answer E

Solution Solution 1

Let x be the total amount of votes casted. From the chart, Brenda received 30% of the votes and had 36 votes. We can express this relationship as $\frac{30}{100}x = 36$. Solving for x , we get $x = \boxed{\text{(E)} 120}$.

Solution 2

We're being asked for the total number of votes cast -- that represents 100% of the total number of votes. Brenda received 36 votes, which is $\frac{30}{100} = \frac{3}{10}$ of the total number of votes. Multiplying 36 by $\frac{10}{3}$, we get the total number of votes, which is $\boxed{\text{(E)} 120}$.

3 What is the value of the expression $\sqrt{16\sqrt{8\sqrt{4}}}$?

- A. 4 B. $4\sqrt{2}$ C. 8 D. $8\sqrt{2}$ E. 16

Answer C

Solution $\sqrt{16\sqrt{8\sqrt{4}}} = \sqrt{16\sqrt{8 \cdot 2}} = \sqrt{16\sqrt{16}} = \sqrt{16 \cdot 4} = \sqrt{64} = \boxed{\text{(C)} 8}$.

4 When 0.000315 is multiplied by 7,928,564 the product is closest to which of the following?

- A. 210 B. 240 C. 2100 D. 2400 E. 24000

Answer D

Solution We can approximate 7,928,564 to 8,000,000 and 0.000315 to 0.0003. Multiplying the two yields 2400. Thus, it shows our answer is **(D) 2400**.

5 What is the value of the expression $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8}{1 + 2 + 3 + 4 + 5 + 6 + 7 + 8}$?

A. 1020

B. 1120

C. 1220

D. 2240

E. 3360

Answer B

Solution **Solution 1**

It is well known that the sum of all numbers from 1 to n is $\frac{n(n+1)}{2}$. Therefore, the denominator is equal to $\frac{8 \cdot 9}{2} = 4 \cdot 9 = 2 \cdot 3 \cdot 6$. Now, we can cancel the factors of 2, 3, and 6 from both the numerator and denominator, only leaving $8 \cdot 7 \cdot 5 \cdot 4 \cdot 1$. This evaluates to **(B) 1120**.

Solution 2

First, we evaluate $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8$ to get 36. We notice that 36 is 6 squared, so we can factor the denominator like $\frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7 \cdot 8}{6 \cdot 6}$ then cancel the 6s out to get $\frac{4 \cdot 5 \cdot 7 \cdot 8}{1}$. Now that we have escaped fraction form, we multiply $4 \cdot 5 \cdot 7 \cdot 8$. Multiplying these, we get **(B) 1120**.

6 If the degree measures of the angles of a triangle are in the ratio 3 : 3 : 4, what is the degree measure of the largest angle of the triangle?

A. 18

B. 36

C. 60

D. 72

E. 90

Answer D

Solution Solution 1

The sum of the ratios is 10. Since the sum of the angles of a triangle is 180° , the ratio can be scaled up to $54 : 54 : 72$ ($3 \cdot 18 : 3 \cdot 18 : 4 \cdot 18$). The numbers in the ratio $54 : 54 : 72$ represent the angles of the triangle. The question asks for the largest, so the answer is **(D) 72**.

Solution 2

We can denote the angles of the triangle as $3x$, $3x$, $4x$. Due to the sum of the angles in a triangle, $3x + 3x + 4x = 180^\circ \implies x = 18^\circ$. The greatest angle is $4x$ and after substitution we get **(D) 72**.

7 Let Z be a 6-digit positive integer, such as **247247**, whose first three digits are the same as its last three digits taken in the same order. Which of the following numbers must also be a factor of Z ?

- A. 11 B. 19 C. 101 D. 111 E. 1111

Answer A

Solution Solution 1

To check, if a number is divisible by 19, take its unit digit and multiply it by 2, then add the result to the rest of the number, and repeat this step until the number is reduced to two digits. If the result is divisible by 19, then the original number is also divisible by 19.

After we got 19 eliminated, we can see that the other options have a lot of 1's in them. The divisibility rule for 11 is add alternating digits up, then take the difference of them. The example number works like that. If we add variables, $ABCDEF$ to make number $ABCABC$, we can see that $(A + C + B) - (B + A + C) = 0$. Which is divisible by 11, so our answer choice is **(A) 11**.

Solution 2

We are given one of the numbers that can represent Z , so we can just try out the options to see which one is a factor of **247247**. By the previous solution, 19 does not work even though it is a factor of the example number. We get **(A) 11**.

8 Malcolm wants to visit Isabella after school today and knows the street where she lives but doesn't know her house number. She tells him, "My house number has two digits, and exactly three of the following four statements about it are true."

- (1) It is prime.
- (2) It is even.
- (3) It is divisible by 7.
- (4) One of its digits is 9.

This information allows Malcolm to determine Isabella's house number. What is its units digit?

- A. 4 B. 6 C. 7 D. 8 E. 9

Answer D

Solution Notice that (1) cannot be true. Otherwise, the number would have to be prime and be either even or divisible by 7. This only happens if the number is 2 or 7, neither of which are two-digit numbers, so we run into a contradiction. Thus, we must have (2), (3), and (4) be true. By (2), the 2-digit number is even, and thus, the digit in the tens place must be 9. The only even 2-digit number starting with 9 and divisible by 7 is 98, which has a units digit of **(D) 8**.

9 All of Marcy's marbles are blue, red, green, or yellow. One third of her marbles are blue, one fourth of them are red, and six of them are green. What is the smallest number of yellow marbles that Marcy could have?

- A. 1 B. 2 C. 3 D. 4 E. 5

Answer D

Solution Solution 1

The 6 green marbles and yellow marbles form $1 - \frac{1}{3} - \frac{1}{4} = \frac{5}{12}$ of the total marbles. Now, suppose the total number of marbles is x . We know the number of yellow marbles is $\frac{5}{12}x - 6$ and a positive integer. Therefore, 12 must divide x . Trying the smallest multiples of 12 for x , we see that when $x = 12$, we get there are -1 yellow marbles, which is impossible. However when $x = 24$, there are $\frac{5}{12} \cdot 24 - 6 = \boxed{\text{(D)} 4}$ yellow marbles, which must be the smallest possible.

Solution 2

Since $\frac{1}{3}$ of the marbles are blue and $\frac{1}{4}$ are red, it is clear that the total number of marbles must be divisible by 12. If there are 12 marbles, then 4 are blue, 3 are red, and 6 are green, meaning that there are -1 yellow marbles. This is impossible. Trying the next multiple of 12, 24, we find that 8 are green, 6 are red, and 6 are green, meaning that the minimum number of yellow marbles is $\boxed{\text{(D)} 4}$.

10 A box contains five cards, numbered 1, 2, 3, 4, and 5. Three cards are selected randomly without replacement from the box. What is the probability that 4 is the largest value selected?

- A. $\frac{1}{10}$ B. $\frac{1}{5}$ C. $\frac{3}{10}$ D. $\frac{2}{5}$
 E. $\frac{1}{2}$

Answer C

Solution Solution 1

There are $\binom{5}{3}$ possible groups of cards that can be selected. If 4 is the largest card selected, then the other two cards must be either 1, 2, or 3, for a total $\binom{3}{2}$ groups of cards.

Then, the probability is just $\frac{\binom{3}{2}}{\binom{5}{3}} = \boxed{\text{(C)} \frac{3}{10}}$.

Solution 2

$P(\text{no } 5) = \frac{4}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} = \frac{2}{5}$. This is the fraction of total cases with no fives. $p(\text{no } 4 \text{ and no } 5) = \frac{3}{5} \cdot \frac{2}{4} \cdot \frac{1}{3} = \frac{6}{60} = \frac{1}{10}$. This is the intersection of no fours and no fives. Subtract the fraction of no fours and no fives from that of no fives. $\frac{2}{5} - \frac{1}{10} = \frac{3}{10} = \boxed{(C) \frac{3}{10}}$.

11 A square-shaped floor is covered with congruent square tiles. If the total number of tiles that lie on the two diagonals is **37**, how many tiles cover the floor?

- A. 148 B. 324 C. 361 D. 1296 E. 1369

Answer C

Solution **Solution 1**

Since the number of tiles lying on both diagonals is **37**, counting one tile twice, there are $37 = 2x - 1 \implies x = 19$ tiles on each side, where x is the number of tiles on the side length of the square. This is because the number of tiles on the square's diagonal is equal to the number of tiles on the square's side length. Therefore, our answer is $19^2 = \boxed{(C) 361}$.

Solution 2

Visualize it as 4 separate diagonals connecting to one square in the middle. Each square on the diagonal corresponds to one square of horizontal/vertical distance (because it's a square). So, we figure out the length of each separate diagonal, multiply by two, and then add 1. (Realize that we can just join two of the separate diagonals on opposite sides together to save some time in calculations.) Therefore, the edge length is:

$$\frac{37 - 1}{4} \cdot 2 + 1 = 19$$

Thus, our solution is $19^2 = \boxed{(C) 361}$.

12 The smallest positive integer greater than 1 that leaves a remainder of 1 when divided by 4, 5, and 6 lies between which of the following pairs of numbers?

- A. 2 and 19 B. 20 and 39
 C. 40 and 59 D. 60 and 79

E. 80 and 124

Answer D

Solution **Solution 1**

Since the remainder is the same for all numbers, then we will only need to find the lowest common multiple of the three given numbers, and add the given remainder. The $\text{LCM}(4, 5, 6)$ is 60. Since $60 + 1 = 61$, that is in the range of **(D) 60 and 79**.

Solution 2

Call the number we want to find n . We can say that

$$n \equiv 1 \pmod{4}$$

$$n \equiv 1 \pmod{5}$$

$$n \equiv 1 \pmod{6}$$

We can also say that $n - 1$ is divisible by 4, 5, and 6. Therefore, $n - 1 = \text{LCM}(4, 5, 6) = 60$, so $n = 60 + 1 = 61$ which is in the range of **(D) 60 and 79**.

13 Peter, Emma, and Kyler played chess with each other. Peter won 4 games and lost 2 games.

Emma won 3 games and lost 3 games. If Kyler lost 3 games, how many games did he win?

A. 0

B. 1

C. 2

D. 3

E. 4

Answer B

Solution Given n games, there must be a total of n wins and n losses. Hence, $4 + 3 + K = 2 + 3 + 3$ where K is Kyler's wins. $K = 1$, so our final answer is **(B) 1**.

14 Chloe and Zoe are both students in Ms. Demeanor's math class. Last night, they each solved half of the problems in their homework assignment alone and then solved the other half together. Chloe had correct answers to only 80% of the problems she solved alone, but overall

88% of her answers were correct. Zoe had correct answers to 90% of the problems she solved alone. What was Zoe's overall percentage of correct answers?

- A. 89 B. 92 C. 93 D. 96 E. 98

Answer C

Solution **Solution 1**

Let the number of questions that they solved alone be x . Let the percentage of problems they correctly solve together be $a\%$. As given,

$$\frac{80x}{100} + \frac{ax}{100} = \frac{2 \cdot 88x}{100}.$$

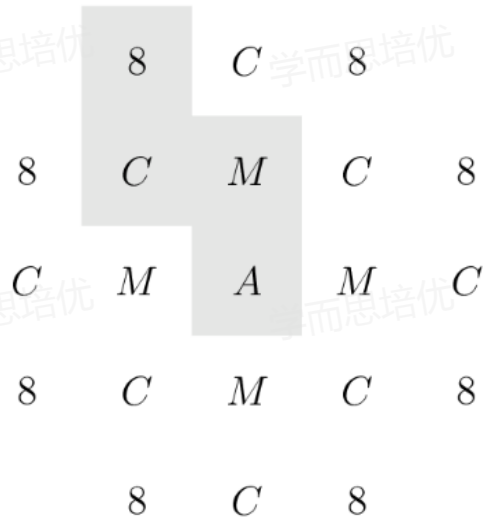
Hence, $a = 96$.

Zoe got $\frac{90x}{100} + \frac{ax}{100} = \frac{186x}{100}$ problems right out of $2x$. Therefore, Zoe got $\frac{\frac{186x}{100}}{2x} = \frac{93}{100} = \boxed{(C) 93}$ percent of the problems correct.

Solution 2

Assume the total amount of problems is 100 per half homework assignment since we are dealing with percentages, not values. Then, we know that Chloe got 80 problems correct by herself and got 176 problems correct overall. We also know that Zoe had 90 problems she did correctly alone. We can see that the total amount of correct problems Chloe and Zoe did together was $176 - 80 = 96$. Therefore, Zoe did $96 + 90 = 186$ problems out of 200 problems correctly. This is $\boxed{(C) 93}$ percent.

- 15 In the arrangement of letters and numerals below, by how many different paths can one spell AMC8? Beginning at the A in the middle, a path only allows moves from one letter to an adjacent (above, below, left, or right, but not diagonal) letter. One example of such a path is traced in the picture.



A. 8

B. 9

C. 12

D. 24

E. 36

Answer D

Solution **Solution 1**

Notice that the **A** is adjacent to 4 **M**s, each **M** is adjacent to 3 **C**s, and each **C** is adjacent to 2 8's. So for each **A**, there are 4 **M**s, and for each **M**, there are 3 **C**s, and for each **C**, there are 2 8s. Thus, the answer is $1 \cdot 4 \cdot 3 \cdot 2 = \boxed{\text{(D) } 24}$.

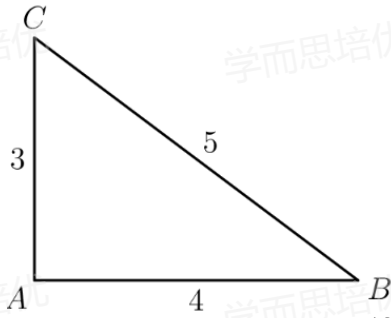
Solution 2

There are three different kinds of paths that are on this diagram. The first kind is when you directly count **A**, **M**, **C** in a straight line. The second is when you count **A**, turn left or right to get **M**, then go up or down to count 8 and **C**. The third is the one where you start with **A**, move up or down to count **M**, turn left or right to count **C**, then move straight again to get 8. There are 8 paths for each kind of path, making for $8 \cdot 3 = \boxed{\text{(D) } 24}$ paths.

16

In the figure below, choose point **D** on $\overline{BC}$ so that $\triangle ACD$ and $\triangle ABD$ have equal perimeters.

What is the area of $\triangle ABD$?



A. $\frac{3}{4}$

B. $\frac{3}{2}$

C. 2

D. $\frac{12}{5}$

E. $\frac{5}{2}$

Answer D

Solution **Solution 1**

We know that the perimeters of the two small triangles are $3 + CD + AD$ and $4 + BD + AD$.

Setting both equal and using $BD + CD = 5$, we have $BD = 2$ and $CD = 3$. Now, we

simply have to find the area of $\triangle ABD$. Since $\frac{BD}{CD} = \frac{2}{3}$, we must have $\frac{[ABD]}{[ACD]} = \frac{2}{3}$.

Combining this with the fact that $[ABC] = [ABD] + [ACD] = \frac{3 \cdot 4}{2} = 6$, we get

$$[ABD] = \frac{2}{5}[ABC] = \frac{2}{5} \cdot 6 = \boxed{\text{(D)} \frac{12}{5}}.$$

Solution 2

Since $\overline{AC}$ is 1 less than $\overline{BC}$, $\overline{CD}$ must be 1 more than $\overline{BD}$ to equate the perimeter. Hence,

$\overline{BD} + \overline{BD} + 1 = 5$, so $\overline{BD} = 2$. Therefore, the area of $\triangle ABD$ is

$$\frac{(2)(4)(\sin B)}{2} = 4\left(\frac{3}{5}\right) = \boxed{\text{(D)} \frac{12}{5}}.$$

- 17 Starting with some gold coins and some empty treasure chests, I tried to put 9 gold coins in each treasure chest, but that left 2 treasure chests empty. So instead I put 6 gold coins in each treasure chest, but then I had 3 gold coins left over. How many gold coins did I have?

A. 9

B. 27

C. 45

D. 63

E. 81

Answer C

Solution **Solution 1**

We can represent the amount of gold with g and the amount of chests with c . We can use the problem to make the following equations:

$$9c - 18 = g$$

$$6c + 3 = g$$

We do this because for 9 chests there are 2 empty and if 9 were in each 9 multiplied by 2 is 18 left.

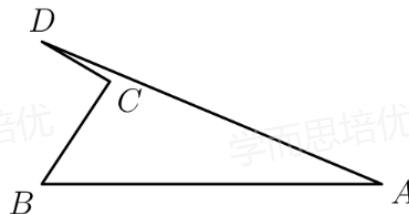
Therefore, $6c + 3 = 9c - 18$. This implies that $c = 7$. We therefore have $g = 45$. So, our answer is **(C) 45**.

Solution 2

With 9 coins, there are $\frac{9}{9} + 2 = 1 + 2 = 3$ chests, by the first condition. These don't fit in with the second condition, so we move onto 27 coins. By the same first condition, there are 5 chests ($\frac{27}{9} + 2$). This also doesn't fit with the second condition. So, onto 45 coins. The first condition implies that there are $\frac{45}{9} + 2 = 7$ chests, which DOES fit with the second condition, since $6 \cdot 7 + 3 = 42 + 3 = 45$. Thus, the desired value is **(C) 45**.

18

In the non-convex quadrilateral $ABCD$ shown below, $\angle BCD$ is a right angle, $AB = 12$, $BC = 4$, $CD = 3$, and $AD = 13$. What is the area of quadrilateral $ABCD$?



A. 12

B. 24

C. 26

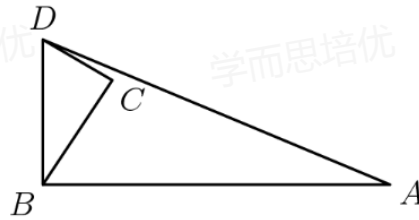
D. 30

E. 36

Answer B

Solution Solution 1

We first connect point B with point D .



We can see that $\triangle BCD$ is a $3 - 4 - 5$ right triangle. We can also see that $\triangle BDA$ is a right triangle, by the $5 - 12 - 13$ Pythagorean triple. With these lengths, we can solve the problem. The area of $\triangle BDA$ is $\frac{5 \cdot 12}{2}$, and the area of $\triangle BCD$ is $\frac{3 \cdot 4}{2}$. Thus, the area of quadrilateral $ABCD$ is $30 - 6 = \boxed{\text{(B) } 24}$.

Solution 2

$\triangle BCD$ is a $3 - 4 - 5$ right triangle. So the area of $\triangle BCD$ is 6. Then we can use Heron's formula to compute the area of $\triangle ABD$ whose sides have lengths 5, 12, and 13. The area of $\triangle ABD = \sqrt{s(s-5)(s-12)(s-13)}$, where s is the semi-perimeter of the triangle, that is $s = (5 + 12 + 13)/2 = 15$. Thus, the area of $\triangle ABD$ is 30, so the area of $ABCD$ is $30 - 6 = \boxed{\text{(B) } 24}$.

19 For any positive integer M , the notation $M!$ denotes the product of the integers 1 through M .

What is the largest integer n for which 5^n is a factor of the sum $98! + 99! + 100!$?

- A. 23 B. 24 C. 25 D. 26 E. 27

Answer D

Solution Factoring out $98! + 99! + 100!$, we have $98!(1 + 99 + 99 \cdot 100)$, which is $98!(10000)$. Next, $98!$ has $\left\lfloor \frac{98}{5} \right\rfloor + \left\lfloor \frac{98}{25} \right\rfloor = 19 + 3 = 22$ factors of 5. The 19 is because of all the multiples of 5. The 3 is because of all the multiples of 25. Now, 10,000 has 4 factors of 5, so there are a total of $22 + 4 = \boxed{\text{(D) } 26}$ factors of 5.

20 An integer between 1000 and 9999, inclusive, is chosen at random. What is the probability that it is an odd integer whose digits are all distinct?

A. $\frac{14}{75}$
E. $\frac{9}{25}$

B. $\frac{56}{225}$

C. $\frac{107}{400}$

D. $\frac{7}{25}$

Answer B

Solution There are 5 options for the last digit as the integer must be odd. The first digit now has 8 options left (it can't be 0 or the same as the last digit). The second digit also has 8 options left (it can't be the same as the first or last digit). Finally, the third digit has 7 options (it can't be the same as the three digits that are already chosen). Since there are 9,000 total integers, our answer is

$$\frac{8 \cdot 8 \cdot 7 \cdot 5}{9000} = \boxed{\text{(B)} \frac{56}{225}}.$$

21 Suppose a , b , and c are nonzero real numbers, and $a + b + c = 0$. What are the possible value(s) for $\frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|}$?

A. 0

B. 1 and -1

C. 2 and -2

D. 0, 2, and -2

E. 0, 1, and -1

Answer A

Solution There are 2 cases to consider:

Case 1: 2 of a , b , and c are positive and the other is negative. Without loss of generality (WLOG), we can assume that a and b are positive and c is negative. In this case, we have that

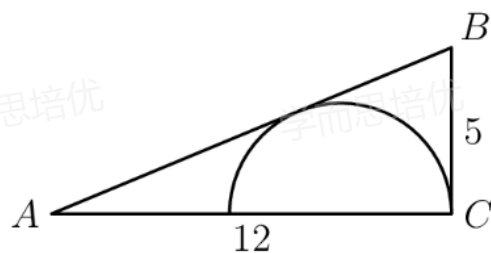
$$\frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|} = 1 + 1 - 1 - 1 = 0.$$

Case 2: 2 of a , b , and c are negative and the other is positive. WLOG, we can assume that a and b are negative and c is positive. In this case, we have that

$$\frac{a}{|a|} + \frac{b}{|b|} + \frac{c}{|c|} + \frac{abc}{|abc|} = -1 - 1 + 1 + 1 = 0.$$

Note these are the only valid cases, for neither 3 negatives nor 3 positives would work as they cannot sum up to 0. In both cases, we get that the given expression equals $\boxed{\text{(A)} 0}$.

- 22 In the right triangle ABC , $AC = 12$, $BC = 5$, and angle C is a right angle. A semicircle is inscribed in the triangle as shown. What is the radius of the semicircle?

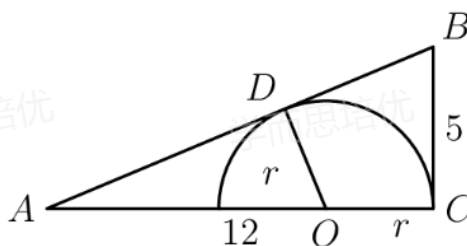


- A. $\frac{7}{6}$ B. $\frac{13}{5}$ C. $\frac{59}{18}$ D. $\frac{10}{3}$
 E. $\frac{60}{13}$

Answer D

Solution Solution 1

We can draw another radius from the center to the point of tangency. This angle, $\angle ODB$, is 90° . Label the center O , the point of tangency D , and the radius r .



Since $ODBC$ is a kite, then $DB = CB = 5$. Also, $AD = 12 - 5 = 7$. By the Pythagorean

Theorem, $r^2 + 7^2 = (12 - r)^2$. Solving, $r^2 + 49 = 144 - 24r + r^2 \Rightarrow 24r = 95 \Rightarrow \boxed{(D) \frac{10}{3}}$.

Solution 2

If we reflect triangle ABC over line AC , we will get isosceles triangle ABD . By the

Pythagorean Theorem, we are capable of finding out that the $AB = AD = 13$. Hence,

$\tan \frac{\angle BAD}{2} = \tan \angle BAC = \frac{5}{12}$. Therefore, as of triangle ABD , the radius of its inscribed circle $r = \frac{\tan \frac{\angle BAD}{2} \cdot (AB + AD - BD)}{2} = \frac{\frac{5}{12} \cdot 16}{2} = \boxed{(D) \frac{10}{3}}$.

Each day for four days, Linda traveled for one hour at a speed that resulted in her traveling one mile in an integer number of minutes. Each day after the first, her speed decreased so that the number of minutes to travel one mile increased by 5 minutes over the preceding day. Each of the four days, her distance traveled was also an integer number of miles. What was the total number of miles for the four trips?

- A. 10 B. 15 C. 25 D. 50 E. 82

Answer C

Solution It is well known that **Distance = Speed · Time**. In the question, we want distance. From the question, we have that the time is 60 minutes or 1 hour. By the equation derived from **Distance = Speed · Time**, we have **Speed = $\frac{\text{Distance}}{\text{Time}}$** , so the speed is 1 mile per x minutes. Because we want the distance, we multiply the time and speed together yielding $60 \text{ mins} \cdot \frac{1 \text{ mile}}{x \text{ mins}}$. The minutes cancel out, so now we have $\frac{60}{x}$ as our distance for the first day. The distance for the following days are:

$$\frac{60}{x}, \frac{60}{x+5}, \frac{60}{x+10}, \frac{60}{x+15}.$$

We know that $x, x+5, x+10, x+15$ are all factors of 60, therefore, $x = 5$ because the factors have to be in an arithmetic sequence with the common difference being 5 and $x = 5$ is the only solution.

$$\frac{60}{5} + \frac{60}{10} + \frac{60}{15} + \frac{60}{20} = 12 + 6 + 4 + 3 = \boxed{(C) 25}.$$

- 24 Mrs. Sanders has three grandchildren, who call her regularly. One calls her every three days, one calls her every four days, and one calls her every five days. All three called her on December 31, 2016. On how many days during the next year did she not receive a phone call from any of her grandchildren?

- A. 78 B. 80 C. 144 D. 146 E. 152

Answer D

Solution **Solution 1**

We use Principle of Inclusion-Exclusion. There are 365 days in the year, and we subtract the days that she gets at least 1 phone call, which is

$$\left\lfloor \frac{365}{3} \right\rfloor + \left\lfloor \frac{365}{4} \right\rfloor + \left\lfloor \frac{365}{5} \right\rfloor.$$

To this result we add the number of days where she gets at least 2 phone calls in a day

because we double subtracted these days, which is

$$\left\lfloor \frac{365}{12} \right\rfloor + \left\lfloor \frac{365}{15} \right\rfloor + \left\lfloor \frac{365}{20} \right\rfloor.$$

We now subtract the number of days where she gets three phone calls, which is $\left\lfloor \frac{365}{60} \right\rfloor$.

Therefore, our answer is

$$365 - \left(\left\lfloor \frac{365}{3} \right\rfloor + \left\lfloor \frac{365}{4} \right\rfloor + \left\lfloor \frac{365}{5} \right\rfloor \right) + \left(\left\lfloor \frac{365}{12} \right\rfloor + \left\lfloor \frac{365}{15} \right\rfloor + \left\lfloor \frac{365}{20} \right\rfloor \right) - \left\lfloor \frac{365}{60} \right\rfloor = 365 - 285 + 7 - 6 = \boxed{\text{(D)} 146}.$$

Solution 2

Note that $\text{lcm}(3, 4, 5) = 60$, so there is a cycle every 60 days.

As shown below, all days in a cycle that Mrs. Sanders receives a phone call from any of her grandchildren are colored in red, yellow, or green.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60

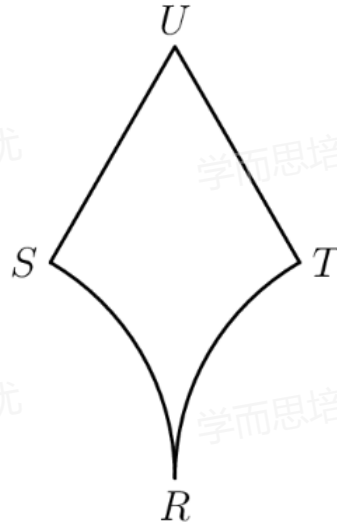
The year 2017 has 365 days, or 6 cycles and 5 days.

- For each cycle, there are 24 days that Mrs. Sanders does not receive a phone call, as indicated by the white squares.

- For the last 5 days, there are 2 days that Mrs. Sanders does not receive a phone call, as indicated by the first 5 days in a cycle. Together, the answer is $24 \cdot 6 + 2 = \boxed{\text{(D)} 146}$.

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In the figure shown, $\overline{US}$ and $\overline{UT}$ are line segments each of length 2, and $m\angle TUS = 60^\circ$. Arcs $\widehat{TR}$ and $\widehat{SR}$ are each one-sixth of a circle with radius 2. What is the area of the region shown?



A. $3\sqrt{3} - \pi$

B. $4\sqrt{3} - \frac{4\pi}{3}$

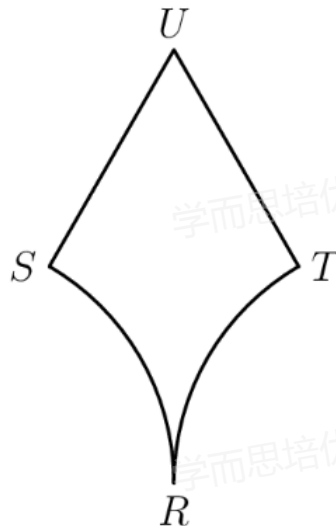
C. $2\sqrt{3}$

D. $4\sqrt{3} - \frac{2\pi}{3}$

E. $4 + \frac{4\pi}{3}$

Answer B

Solution Solution 1



In addition to the given diagram, we can draw lines $\overline{SR}$ and $\overline{RT}$. The area of rhombus

$SRTU$ is half the product of its diagonals, which is $\frac{2\sqrt{3} \cdot 2}{2} = 2\sqrt{3}$. However, we have to

subtract off the circular segments. The area of those can be found by computing the area of the circle with radius 2, multiplying it by $\frac{1}{6}$, then finally subtracting the area of an equilateral

triangle with a side length 2 from the sector. The sum of the areas of the circular segments is $2\left(\frac{4\pi}{6} - \sqrt{3}\right)$. The area of rhombus $SRTU$ minus the circular segments is

$$2\sqrt{3} - \frac{4\pi}{3} + 2\sqrt{3} = \boxed{\text{(B)} 4\sqrt{3} - \frac{4\pi}{3}}.$$

Solution 2

We can extend $\overline{US}$, $\overline{UT}$ to X and Y , respectively, such that X and Y are collinear to point R . Connect $\overline{XY}$. We can see points X , Y are probably circle centers of arc SR , TR ,

respectively. So, $\overline{XS} = 2 = \overline{TY}$. Thus, $\triangle UXY$ is equilateral. The area of $\triangle UXY$ is $\frac{\sqrt{3}}{4} \cdot 4^2$, or $4\sqrt{3}$, and both one sixth circles total up to $\frac{4\pi}{3}$. Finally, the answer is $\boxed{\text{(B)} 4\sqrt{3} - \frac{4\pi}{3}}$.